2. Two brands of cigarettes are compared for nicotine content to determine if a difference exists. Independent and random samples of each brand are taken and summarized below. Find a 99% confidence interval for the true difference in mean nicotine content when we ~~may~~ assume normality.

$\Rightarrow \mu_1 \neq \mu_2$

$\mu_1 - \mu_2 \neq 0$

	(1) Brand A	(2) Brand B
Sample size	18	11
Sample mean	26.6 mg	32.9 mg
Sample standard deviation (s)	4.7 mg	4.4 mg

$\frac{S_1}{S_2} = \frac{4.7}{4.4} = 1.07$

$\sigma_1^2 = \sigma_2^2 = \sigma^2$

S_p^2 estimates σ^2

99% CI for the true difference in mean nicotine content: $(\mu_1 - \mu_2)$

Random Variable: X = nicotine content (mg)

Assumptions: SRS

n_1 and n_2 are independently sampled.

S_1^2 and S_2^2 are good estimates of σ_1^2 and σ_2^2

can assume equal population variances

$(\bar{x}_1 - \bar{x}_2)$ is normal because nicotine contents are normal.

point estimate ± rel coeff * std error

$$(\bar{x}_1 - \bar{x}_2) \pm t_{n_1+n_2-2,\alpha/2} * \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$$

$$(26.6 - 32.9) \pm 2.771 * \sqrt{\frac{21.08}{18} + \frac{21.08}{11}}$$

$$-6.3 \pm 4.87$$

ReliabilityCoefficient: $t_{27} = \pm 2.771$

(−11.17 mg , −1.43 mg)

∅ is not in CI

S_p^2 = pooled variance

$$S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

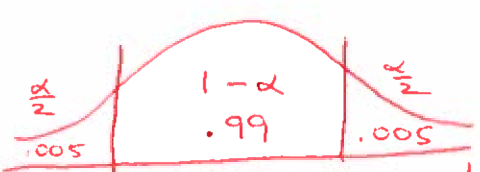
$$S_p^2 = \frac{(17)(4.7)^2 + (10)(4.4)^2}{27}$$

$$S_p^2 = 21.08$$

H₀: $\mu_1 - \mu_2 = 0$

Hₐ: $\mu_1 - \mu_2 \neq 0$

Conclusion: Reject H₀, 99% sure there is a difference in mean nicotine content between Brands A and B.



$t_{n_1+n_2-2} = t_{27}$

invt(.005, 27)

3. Many students believe that coffee machine B dispenses more coffee/cup (oz), on average, when compared to coffee machine A. To test this belief, random and independent samples were taken from each coffee machine with the following results.

$\mu_B > \mu_A$

$\mu_2 > \mu_1$

	(1) Machine A	(2) Machine B
Sample size	10	12
Sample mean	5.38 oz	5.92 oz
Sample standard deviation (s)	1.89 oz	0.83 oz

$\frac{s_1}{s_2} = \frac{1.89}{0.83} = 2.277$

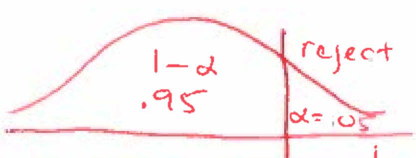
Test the students' claim at the 5% significance level. Assume normality.

$\mu_2 > \mu_1$ $\quad$ $\mu_2 > \mu_1$

$0 > \mu_1 - \mu_2$ $\quad$ $\mu_2 - \mu_1 > 0$

$$t_{obs} = \frac{(\bar{x}_2 - \bar{x}_1) - (\cancel{\mu_2 - \mu_1})^{0}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(5.92 - 5.38)}{\sqrt{\frac{(1.89)^2}{10} + \frac{(0.83)^2}{12}}}$$



$t_{df'} = t_9$

invt(.95, 9) = 1.833

df' = smaller sample size − 1

t_9

$t_{obs} = 0.84$ $\quad$ tcdf(0.84, ∞, 9)

Random Variable: X = amount of coffee/cup (oz)

Assumptions: SRS

n_1 and n_2 are independently sampled

s_1^2 and s_2^2 are good estimates of σ_1^2 and σ_2^2

can not assume equal population variance

$(\bar{x}_1 - \bar{x}_2)$ is normal because amounts of coffee/cup are normal

H_0: $\mu_2 - \mu_1 = 0$

H_a: $\mu_2 - \mu_1 > 0$

Test Statistic: $t_{obs} = 0.84$

Rejection Region: Reject H_0 if $t_{obs} \geq 1.833$

Conclusion: Fail to reject H_0, < 95% sure we couldn't ~~prove~~ prove the mean amount of coffee/cup is more from Machine B than Machine A.

P-Value: 0.211

What type of error could you have subjected yourself to and explain what this means in terms of this situation?

Type II error − Machine B does give more coffee/cup than Machine A, but we couldn't prove it.

Two Population Means

1. Makers of two small cars, A and B, each claim that their automobile gets superior gas mileage on the highway. To decide the matter once and for all, the makers of car A decide to run a carefully controlled experiment designed to prove that the mean number of miles per gallon on the highway obtained by car A is greater than that of car B. Twenty-five cars of type A and twenty-four cars of type B were randomly and independently sampled. The sample mean mileage for car A was 37.5 mpg and for car B was 36.2 mpg. The respective sample variances were 3.6 mpg^2 and 2.5 mpg^2 for car A and B respectively. Assuming normality, test at the 5% significance level to determine if car A gets more miles per gallon on the highway compared to car B.

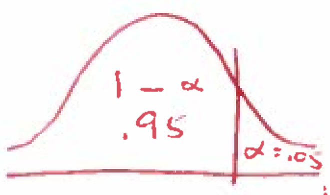
	(1) A	(2) B
n	25	24
$\bar{x}$	37.5	36.2
s^2	3.6	2.5

$\frac{s_1}{s_2} = \frac{\sqrt{3.6}}{\sqrt{2.5}} = 1.2$

$\mu_A > \mu_B$

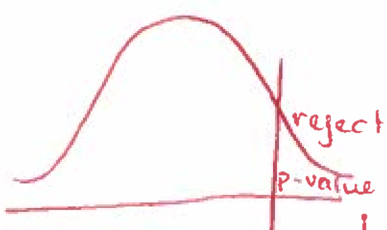
$$t_{obs} = \frac{(\bar{x}_1 - \bar{x}_2) - \underbrace{(\mu_1 - \mu_2)}_{\to 0}}{\sqrt{\frac{Sp^2}{n_1} + \frac{Sp^2}{n_2}}}$$

$$= \frac{(37.5 - 36.2)}{\sqrt{\frac{3.06}{25} + \frac{3.06}{24}}}$$



t_{47}

invt(.95, 47) = 1.678



$t_{obs} = 2.6$ t_{47}

tcdf(2.6, ∞, 47) = 0.00621

$$Sp^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$Sp^2 = \frac{(24)(3.6) + (23)(2.5)}{47}$$

$Sp^2 = 3.06$

Random Variable mileage rate (mpg)

Assumptions: SRS

n_1 and n_2 are independently sampled

s_1^2 and s_2^2 are good estimates of σ_1^2 and σ_2^2 respectively

$(\bar{x}_1 - \bar{x}_2)$ is normally distributed because mileage rates are normal

equal population variances

H_0 $\mu_1 - \mu_2 = 0$

H_a $\mu_1 - \mu_2 > 0$

Test Statistic $t_{obs} = 2.6$

Rejection Region Reject H_0 if $t_{obs} \geq 1.678$

Conclusion Reject H_0, 95% sure the mean mileage rate for car A is more than car B.

P-value 0.00621

2. A manager is interested in determining if there is a difference in profits during store hours. Two different schedules are under consideration. They are: Schedule I (9:00-5:00) and Schedule II (10:00-6:00). There is no preconceived notion as to whether a difference exists or not; the problem is to estimate the difference in mean sales for the two schedules via a 95% confidence interval. Over a period of time, independent and random samples are taken and the following sample data are obtained on sales ($) using these two schedules.

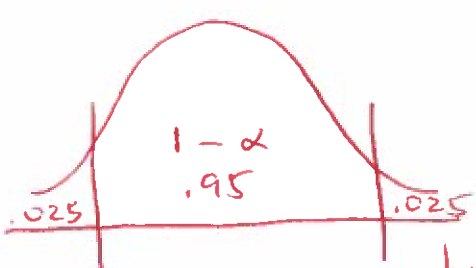
	(1) Schedule I	(2) Schedule II
Sample Size	15	25
Sample Mean	$570	$600
Sample Standard Deviation	$ 85	$ 22

$\frac{s_1}{s_2} = \frac{85}{22} = 3.9$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{df', \alpha/2} * \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(570-600) \pm 2.145 * \sqrt{\frac{85^2}{15} + \frac{22^2}{25}}$$

$$-30 \pm 48.01$$



$t_{df'} = t_{14}$

invt(.025,14) = −2.145

Random Variable: profit ($)

Assumptions: SRS

n_1 and n_2 are independently sampled

s_1^2 and s_2^2 are good estimates of σ_1^2 and σ_2^2 respectively

$(\bar{x}_1 - \bar{x}_2)$ is normally distributed because profits are normal

can not assume equal population variances

Reliability Coefficient: $t_{14} = \pm 2.145$

(−$78.01 , $18.01)

H_0: $\mu_1 - \mu_2 = 0$

H_a: $\mu_1 - \mu_2 \neq 0$

0 is in CI

Conclusion: Fail to reject H_0 < 95% sure we couldn't prove a difference in mean profits between schedules I and II.

3. A sports-shoe manufacturer claims that joggers who wear its brand of shoe will jog faster than those who don't. A sample of seven joggers is taken, and they agree to test the claim on a 1-mile track. The rates (in minutes) of the joggers while wearing the manufacturer shoes one day and then the next day while wearing any other brand of shoe are shown here. Assuming normality, test the claim at the 5% significance level.

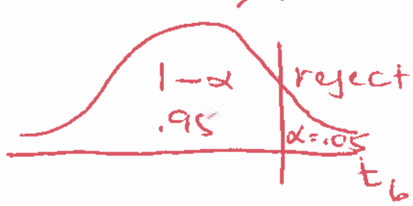
Runner	1	2	3	4	5	6	7
Manufacturer's	8.2	6.3	9.2	8.6	6.8	8.7	8.0
Other	9.1	6.8	9.8	8.0	5.8	10.0	8.4
d_i O − M	0.9	0.5	0.6	−0.6	−1.0	1.3	0.4

one observational unit → 2 measurements → dependent sampling

$\bar{d} = 0.3$ $s_d = 0.82$ $n = 7$

* jog faster in manufacturer's shoes

$$t_{obs} = \frac{\bar{d} - \cancel{\mu_d}^{\,0}}{s_d/\sqrt{n}} = \frac{0.3}{0.82/\sqrt{7}}$$



invt(.95, 6) = 1.943

tcdf(0.97, 6) = 0.185

Random Variable jogging rate (min)

Assumptions: SRS

n_1 and n_2 are dependently sampled

s_d is a good estimate of σ_d

$\bar{d}$ is normally distributed because jogging rates are normal

H_0 $\mu_d \le 0$

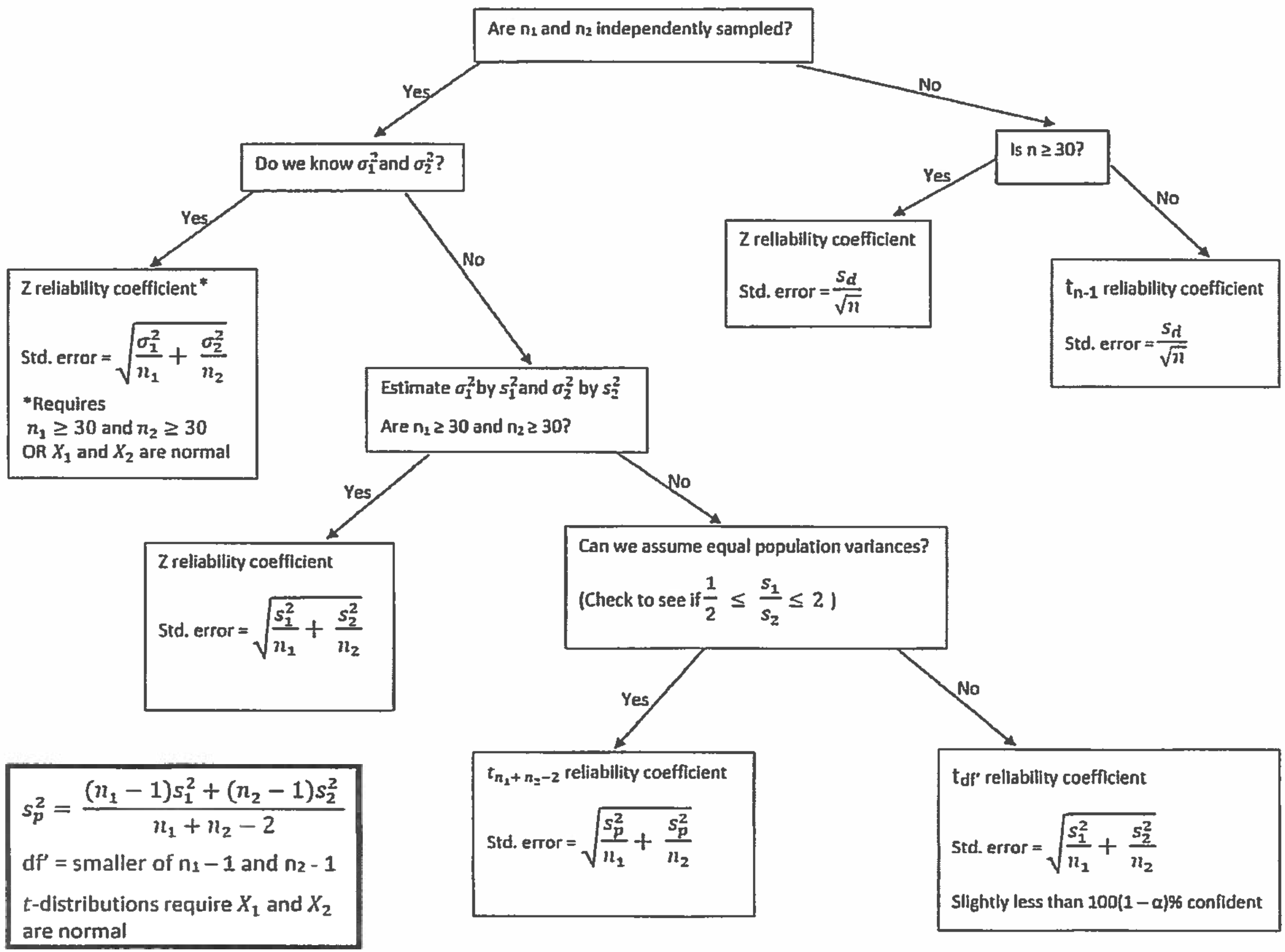
H_a $\mu_d > 0$

Test Statistic $t_{obs} = 0.97$

Rejection Region Reject H_0 if $t_{obs} \ge 1.943$

Conclusion Fail to reject H_0. 95% sure we couldn't prove a runner jogs faster, on average, in the manufacturer's shoes compared to any other brand

P-value 0.185



Are n₁ and n₂ independently sampled?
Yes
No
Do we know σ_1^2 and σ_2^2?
Yes
No
Z reliability coefficient*
Std. error = $\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$
*Requires
$n_1 \geq 30$ and $n_2 \geq 30$
OR X_1 and X_2 are normal
Estimate σ_1^2 by s_1^2 and σ_2^2 by s_2^2
Are $n_1 \geq 30$ and $n_2 \geq 30$?
Yes
No
Z reliability coefficient
Std. error = $\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$
Can we assume equal population variances?
(Check to see if $\frac{1}{2} \leq \frac{s_1}{s_2} \leq 2$)
Yes
No
$t_{n_1+n_2-2}$ reliability coefficient
Std. error = $\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}$
$t_{df'}$ reliability coefficient
Std. error = $\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$
Slightly less than $100(1-\alpha)$% confident
Is $n \geq 30$?
Yes
No
Z reliability coefficient
Std. error = $\frac{s_d}{\sqrt{n}}$
t_{n-1} reliability coefficient
Std. error = $\frac{s_d}{\sqrt{n}}$
$s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$
df' = smaller of $n_1 - 1$ and $n_2 - 1$
t-distributions require X_1 and X_2 are normal

Two Population Means -Dependent Sampling

Confidence Interval:

point estimate $\pm$ reliability coefficient * standard error

$(\bar{d}) \pm$ (from the flowchart)

Hypothesis Test:

1. Statement of Hypothesis

$H_o: \mu_d = 0$	$H_o: \mu_d = 0$	$H_o: \mu_d = 0$
$H_a: \mu_d \neq 0$	$H_a: \mu_d > 0$	$H_a: \mu_d < 0$

2. Test Statistic: Distribution comes from the flowchart

$$\text{Test statistic} = \frac{\text{point estimate} - \text{parameter}}{\text{standard error}}$$

$$= \frac{\bar{d} - \mu_d}{\text{(from flowchart)}}$$

3. Rejection Region(s): same as before

4. Conclusion: same as before

5. P-value: same as before

Dependent Sampling From Two Populations

1. Twelve ranch-style houses were divided into six pairs on the basis of square footage and exposure to the sun. Awnings were installed on one house in each pair. Summarized below are the mean electric bills for each house during the three hottest months of the summer. Construct a 99% confidence interval to determine if there is a difference in electric bills between homes with and without an awning.

House Pair	Without Awning	With Awning	$d_i = w/o - w$
1	$110	$ 98	12
2	$ 86	$ 90	-4
3	$125	$114	11
4	$105	$109	-4
5	$ 78	$ 74	4
6	$145	$141	4

$\bar{d} = 3.83$

$s_d = 6.94$

$n = 6$

Random Variable: X = electric bill ($)

Assumptions: SRS

n_1 and n_2 are dependently sampled

s_d is a good estimate of σ_d

$\bar{d}$ is normally distributed because electric bills are normal

point estimate ± rel coeff * std error

$\bar{d} \pm t_{n-1,\alpha/2} * s_d/\sqrt{n}$

$3.83 \pm t_{5,.005} * 6.94/\sqrt{6}$

$3.83 \pm 4.032 * 6.94/\sqrt{6}$

3.83 ± 11.42

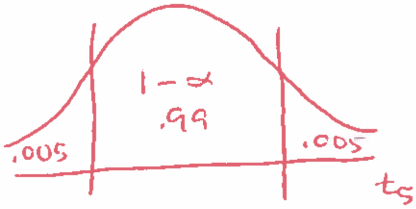
Reliability Coefficient: $t_5 = \pm 4.032$

(-$7.59 , $15.25)

H_0: $\mu_d = 0$

H_a: $\mu_d \neq 0$

Ø is in CI



invt(.005,5) = -4.032

Conclusion: Fail to reject H_0, 99% sure we couldn't prove electric bills, on average, are different for homes with awnings compared to those without.

2. An article entitled "Fuel and Energy Metabolism in Fasting Humans" compared the fasting glucose levels (nmoles/L) of six volunteers after 12 hours of fasting and after 60 hours of fasting. Assuming normality, do the data suggest at the 5% significance level that glucose levels decrease as fasting increases?

Glucose Levels (nmoles/L)

Subject	12 hours fasting	60 hours fasting	$d_i = 12 - 60$
1	6.5	5.9	0.6
2	7.1	6.9	0.2
3	5.5	5.2	0.3
4	8.0	6.9	1.1
5	6.3	5.9	0.4
6	7.1	6.5	0.6

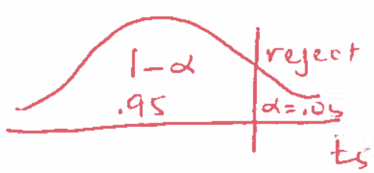
$\bar{d} = 0.53$

$S_d = 0.32$

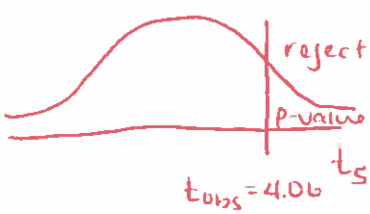
$n = 6$

Random Variable: X = glucose levels (nmoles/L)

$$t_{obs} = \frac{\bar{d} - \cancel{\mu_d}^{\,0}}{S_d/\sqrt{n}} = \frac{0.53}{0.32/\sqrt{6}}$$



invt(.95,5) = 2.015



tcdf(4.06,∞,5)

Assumptions: SRS

n_1 and n_2 are dependently sampled

S_d is a good estimate of σ_d

$\bar{d}$ is normally distributed because glucose levels are normal

H_0: $\mu_d = 0$

H_a: $\mu_d > 0$

Test Statistic: $t_{obs} = 4.06$

Rejection Region: Reject H_0 if $t_{obs} \geq 2.015$

Conclusion: Reject H_0, 95% sure that glucose levels, on average, decrease as fasting increases

P-Value: 0.0049

Would your conclusion have changed if you had tested at the 1% significance level? Explain.

No, p-value (.0049) < α(.01) ⇒ reject H_0

3. A research study looked at 10 subjects and measured their blood pressure (mm Hg) both before and after an exercise program. The researchers wanted to know if the exercise program produced a difference in blood pressures. They computed their difference in pairs as d_i = BEFORE – AFTER

a. What are their hypothesis statements?

H_o: $\mu_d = 0$

H_a: $\mu_d \neq 0$

b. The output for their analysis is as follows. What is their full conclusion?

Variable	N	Mean	StDev	SE Mean	95% C.I.
D	10	6.10	4.79	1.52	(2.67, 9.53)

Ø is outside CI

Conclusion: Reject H_o, 95% sure blood pressures, on average are different before compared to after an exercise program.

4. A paired plot experiment was conducted where the researchers took a field and broke the field up into 10 equal plots and then divided each of these plots in half. Randomly choosing WITHIN each plot, one half got a fertilizer application and the other half did not. They were investigating whether fertilizer increased the yield of sugar beets. The output for their analysis is as follows. Assuming normality, test at the 5% significance level.

Variable	N	Mean	StDev	SE Mean	T	PValue
D	10	3.20	2.741	0.867	3.67	0.0025

① t_{obs} is (+) so right tail test

d_i = fertilizer – no fertilizer

H_o: $\mu_d = 0$

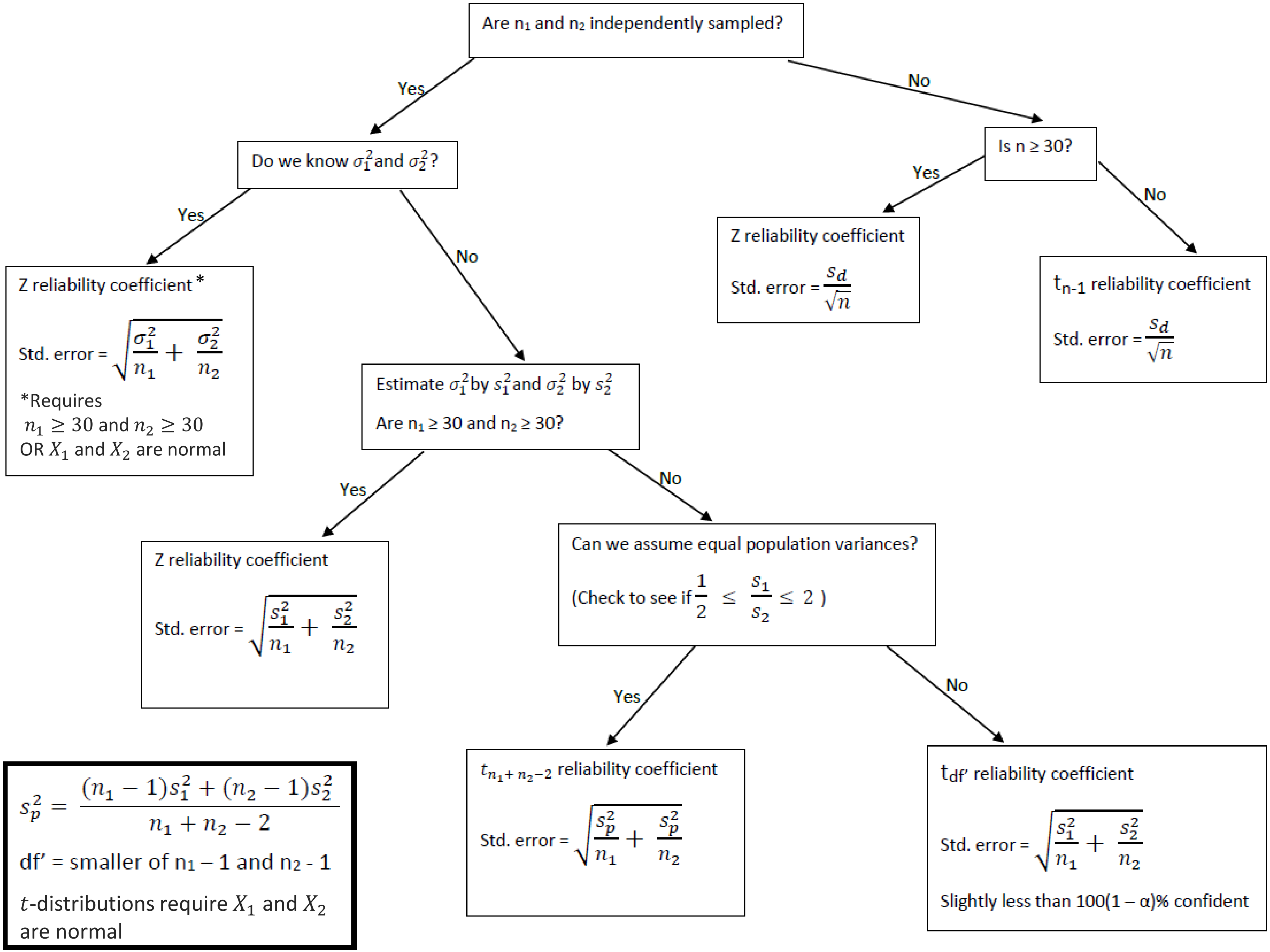
H_a: $\mu_d > 0$

Test Statistic: $t_{obs} = 3.67$

p-value (0.0025) < α (.05)

P-Value: 0.0025

Conclusion: Reject H_o, 95% sure the yield of sugarbeets, on average, increases on plots with fertilizer compared to no fertilizer.



Are n_1 and n_2 independently sampled?
Yes
No
Do we know σ_1^2 and σ_2^2?
Yes
Z reliability coefficient *
Std. error = $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
*Requires
$n_1 \geq 30$ and $n_2 \geq 30$
OR X_1 and X_2 are normal
No
Estimate σ_1^2 by s_1^2 and σ_2^2 by s_2^2
Are $n_1 \geq 30$ and $n_2 \geq 30$?
Yes
Z reliability coefficient
Std. error = $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
No
Can we assume equal population variances?
(Check to see if $\frac{1}{2} \leq \frac{s_1}{s_2} \leq 2$)
Yes
$t_{n_1+n_2-2}$ reliability coefficient
Std. error = $\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$
No
$t_{df'}$ reliability coefficient
Std. error = $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
Slightly less than 100(1 – α)% confident
Is n ≥ 30?
Yes
Z reliability coefficient
Std. error = $\frac{s_d}{\sqrt{n}}$
No
t_{n-1} reliability coefficient
Std. error = $\frac{s_d}{\sqrt{n}}$
$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$
df′ = smaller of $n_1 - 1$ and $n_2 - 1$
t-distributions require X_1 and X_2 are normal

Two Population Means -Dependent Sampling

Confidence Interval:

point estimate $\pm$ reliability coefficient * standard error

$(\bar{d}) \pm$ (from the flowchart)

Hypothesis Test:

1. Statement of Hypothesis

   $H_o: \mu_d = 0$ $\quad$ $H_o: \mu_d = 0$ $\quad$ $H_o: \mu_d = 0$

   $H_a: \mu_d \neq 0$ $\quad$ $H_a: \mu_d > 0$ $\quad$ $H_a: \mu_d < 0$

2. Test Statistic: Distribution comes from the flowchart

   $$\text{Test statistic} = \frac{\text{point estimate} - \text{parameter}}{\text{standard error}}$$

   $$= \frac{\bar{d} - \mu_d}{\text{(from flowchart)}}$$

3. Rejection Region(s): same as before

4. Conclusion: same as before

5. P-value: same as before

Two Population Means -Independent Sampling

Confidence Interval:

point estimate $\pm$ reliability coefficient * standard error

$(\bar{x}_1 - \bar{x}_2) \pm$ (from the flowchart)

Hypothesis Test:

1. Statement of Hypothesis

   $H_o: \mu_1 - \mu_2 = 0$ $\quad$ $H_o: \mu_1 - \mu_2 = 0$ $\quad$ $H_o: \mu_1 - \mu_2 = 0$

   $H_a: \mu_1 - \mu_2 \neq 0$ $\quad$ $H_a: \mu_1 - \mu_2 > 0$ $\quad$ $H_a: \mu_1 - \mu_2 < 0$

2. Test Statistic: Distribution comes from the flowchart

   $$\text{Test statistic} = \frac{\text{point estimate} - \text{parameter}}{\text{standard error}}$$

   $$= \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\text{(from flowchart)}}$$

3. Rejection Region(s): same as before

4. Conclusion: same as before

5. P-value: same as before

Two Population Means

1. Makers of two small cars, A and B, each claim that their automobile gets superior gas mileage on the highway. To decide the matter once and for all, the makers of car A decide to run a carefully controlled experiment designed to prove that the mean number of miles per gallon on the highway obtained by car A is greater than that of car B. Twenty-five cars of type A and twenty-four cars of type B were randomly and independently sampled. The sample mean mileage for car A was 37.5 mpg and for car B was 36.2 mpg. The respective sample variances were 3.6 mpg^2 and 2.5 mpg^2 for car A and B respectively. Assuming normality, test at the 5% significance level to determine if car A gets more miles per gallon on the highway compared to car B.

X:______________________________

Assumptions:______________________________

____

____

____

____

H_o______________________________

H_a______________________________

Test Statistic______________________________

Rejection Region______________________________

Conclusion______________________________

____

____

____

P-value______________________________

2. A manager is interested in determining if there is a difference in profits during store hours. Two different schedules are under consideration. They are: Schedule I (9:00-5:00) and Schedule II (10:00-6:00). There is no preconceived notion as to whether a difference exists or not; the problem is to estimate the difference in mean sales for the two schedules via a 95% confidence interval. Over a period of time, independent and random samples are taken and the following sample data are obtained on sales ($) using these two schedules.

	Schedule I	Schedule II
Sample Size	15	25
Sample Mean	$570	$600
Sample Standard Deviation	$ 85	$ 22

X:____

Assumptions:________________________________

____

____

____

____

Reliability Coefficient_________________________

(____)

H_o____

H_a____

Conclusion__________________________________

____

____

____

3. A sports-shoe manufacturer claims that joggers who wear its brand of shoe will jog faster than those who don't. A sample of seven joggers is taken, and they agree to test the claim on a 1-mile track. The rates (in minutes) of the joggers while wearing the manufacturer shoes one day and then the next day while wearing any other brand of shoe are shown here. Assuming normality, test the claim at the 5% significance level.

Runner	1	2	3	4	5	6	7
Manufacturer's	8.2	6.3	9.2	8.6	6.8	8.7	8.0
Other	9.1	6.8	9.8	8.0	5.8	10.0	8.4

X:______________________________

Assumptions:______________________________

____

____

____

____

H_o______________________________

H_a______________________________

Test Statistic______________________________

Rejection Region______________________________

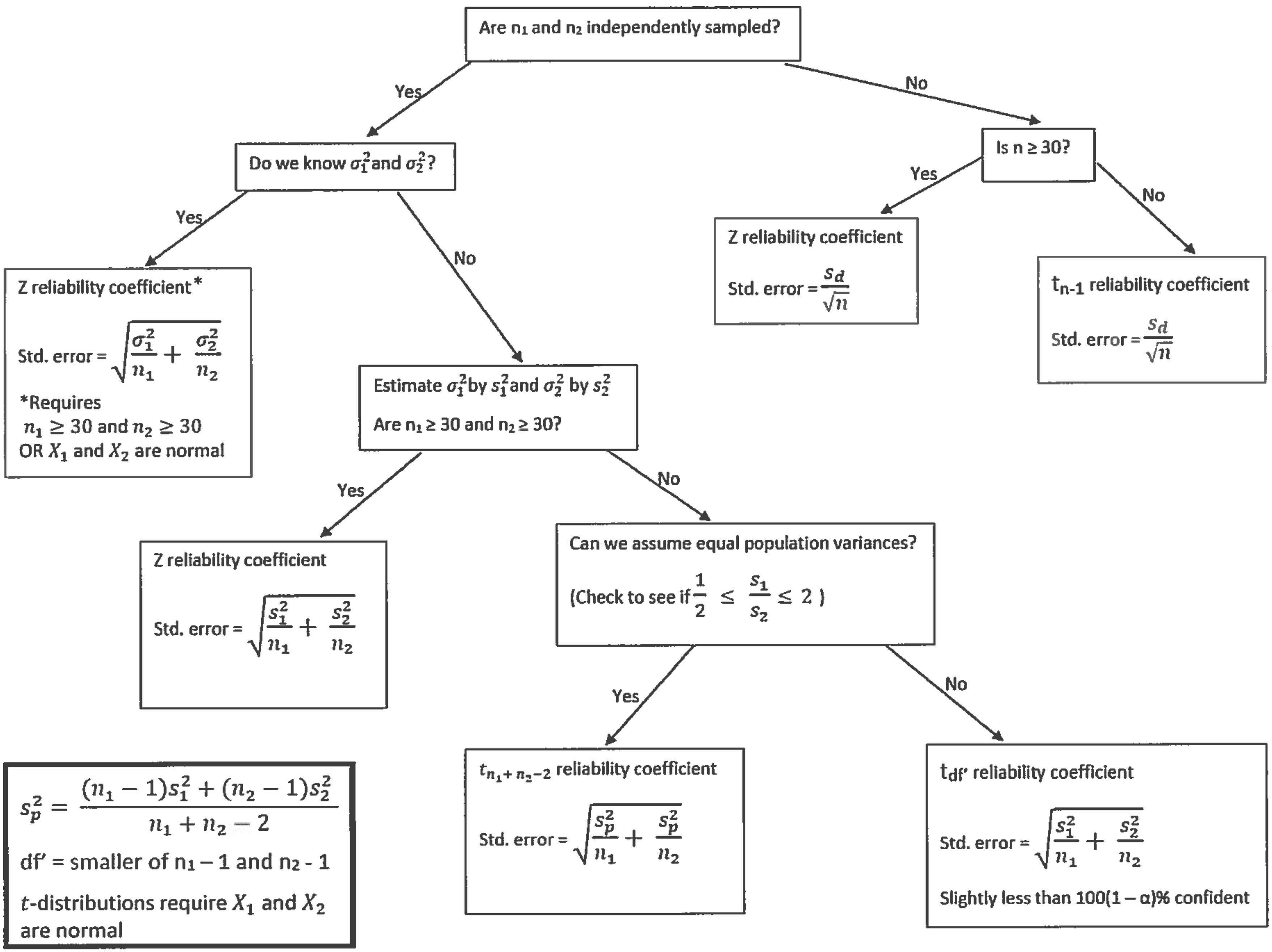
Conclusion______________________________

____

____

____

P-value______________________________


Are n_1 and n_2 independently sampled?
Yes
No
Do we know σ_1^2 and σ_2^2?
Yes
No
Z reliability coefficient*
Std. error = $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
*Requires
$n_1 \geq 30$ and $n_2 \geq 30$
OR X_1 and X_2 are normal
Estimate σ_1^2 by s_1^2 and σ_2^2 by s_2^2
Are $n_1 \geq 30$ and $n_2 \geq 30$?
Yes
No
Z reliability coefficient
Std. error = $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
Can we assume equal population variances?
(Check to see if $\frac{1}{2} \leq \frac{s_1}{s_2} \leq 2$)
Yes
No
$t_{n_1+n_2-2}$ reliability coefficient
Std. error = $\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$
$t_{df'}$ reliability coefficient
Std. error = $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
Slightly less than 100(1 – α)% confident
Is $n \geq 30$?
Yes
No
Z reliability coefficient
Std. error = $\frac{s_d}{\sqrt{n}}$
t_{n-1} reliability coefficient
Std. error = $\frac{s_d}{\sqrt{n}}$
$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$
df' = smaller of $n_1 - 1$ and $n_2 - 1$
t-distributions require X_1 and X_2 are normal

Dependent Sampling From Two Populations

1. Twelve ranch-style houses were divided into six pairs on the basis of square footage and exposure to the sun. Awnings were installed on one house in each pair. Summarized below are the mean electric bills for each house during the three hottest months of the summer. Construct a 99% confidence interval to determine if there is a difference in electric bills between homes with and without an awning.

House Pair	Without Awning	With Awning
1	$110	$ 98
2	$ 86	$ 90
3	$125	$114
4	$105	$109
5	$ 78	$ 74
6	$145	$141

X:____

Assumptions:________________________________

____

____

____

____

Reliability Coefficient:_________________________

(____)

H_o:____

H_a:____

Conclusion:_________________________________

____

____

____

2. An article entitled "Fuel and Energy Metabolism in Fasting Humans" compared the fasting glucose levels (nmoles/L) of six volunteers after 12 hours of fasting and after 60 hours of fasting. Assuming normality, do the data suggest at the 5% significance level that glucose levels decrease as fasting increases?

Glucose Levels (nmoles/L)

Subject	12 hours fasting	60 hours fasting
1	6.5	5.9
2	7.1	6.9
3	5.5	5.2
4	8.0	6.9
5	6.3	5.9
6	7.1	6.5

X:______________________________

Assumptions:__________________________

____

____

____

____

H_o:______________________________

H_a:______________________________

Test Statistic:________________________

Rejection Region:______________________

Conclusion:__________________________

____

____

____

P-Value:____________________________

Would your conclusion have changed if you had tested at the 1% significance level? Explain.

3. A research study looked at 10 subjects and measured their blood pressure (mm Hg) both before and after an exercise program. The researchers wanted to know if the exercise program produced a difference in blood pressures. They computed their difference in pairs as d_i = BEFORE – AFTER

a. What are their hypothesis statements?

H_o:____

H_a:____

b. The output for their analysis is as follows. What is their full conclusion?

Variable	N	Mean	StDev	SE Mean	95% C.I.
D	10	6.10	4.79	1.52	(2.67, 9.53)

Conclusion:________________________________

____

____

____

4. A paired plot experiment was conducted where the researchers took a field and broke the field up into 10 equal plots and then divided each of these plots in half. Randomly choosing WITHIN each plot, one half got a fertilizer application and the other half did not. They were investigating whether fertilizer increased the yield of sugar beets. The output for their analysis is as follows. Assuming normality, test at the 5% significance level.

Variable	N	Mean	StDev	SE Mean	T	PValue
D	10	3.20	2.741	0.867	3.67	0.0025

d_i = ______________________________________

H_o:____

H_a:____

Test Statistic:______________________________

P-Value:_____________________________________

Conclusion:__________________________________

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Estimation and Hypothesis Testing for Two Population Means

1. A restaurant has changed its menu so that most meals now offered cost less than the meals they used to offer. However, the new meals are also less expensive to prepare, and the owner wants to compare the mean net daily income obtained between the two types of meals. Random samples of 50 net daily incomes from the earlier period and 30 net daily incomes from the current period are selected and the sample results are below. Compute a 95% confidence interval for the difference in mean net daily incomes between the two periods so the owner may discern whether a significant difference exists.

Higher Priced Meals	Lower Priced Meals
n = 50	n = 30
$\bar{x}$ = \$185	$\bar{x}$ = \$166
s = \$41	s = \$53

X:____

Assumptions:________________________________

____

____

____

____

ReliabilityCoefficient:_________________________

(____________________,____________________)

H_o:____

H_a:____

Conclusion:__________________________________

____

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____

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2. Two brands of cigarettes are compared for nicotine content to determine if a difference exists. Independent and random samples of each brand are taken and summarized below. Find a 99% confidence interval for the true difference in mean nicotine content when we may assume normality.

	Brand A	Brand B
Sample size	18	11
Sample mean	26.6 mg	32.9 mg
Sample standard deviation	4.7 mg	4.4 mg

X:____

Assumptions:________________________________

____

____

____

____

ReliabilityCoefficient:_________________________

(______________________,__________________)

H_o:____

H_a:____

Conclusion:__________________________________

____

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____

____

3. Many students believe that coffee machine B dispenses more coffee/cup (oz), on average, when compared to coffee machine A. To test this belief, random and independent samples were taken from each coffee machine with the following results.

	Machine A	Machine B
Sample size	10	12
Sample mean	5.38 oz	5.92 oz
Sample standard deviation	1.89 oz	0.83 oz

Test the students' claim at the 5% significance level. Assume normality.

X:____

Assumptions:________________________________

____

____

____

____

H_o:____

H_a:____

Test Statistic:_______________________________

Rejection Region:_____________________________

Conclusion:__________________________________

____

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____

P-Value:_____________________________________

What type of error could you have subjected yourself to and explain what this means in terms of this situation?

4. A sample of 15 one-pound jars of coffee of Brand A showed that the sample mean amount of caffeine in these jars was 80 mg/jar and the sample variance was 25. Another independent sample of 12 one-pound jars of coffee of Brand B gave a mean amount of caffeine equal to 77 mg/jar and a sample variance of 36. Assuming normality, does the data support the claim that Brand B, on average, has less caffeine than Brand A which is what the company is advertising? Test at the 10% significance level.

Brand A Brand B

X:____

Assumptions:________________________________

H_o:____

H_a:____

Test Statistic:________________________________

Rejection Region:______________________________

Conclusion:__________________________________

P-Value:____________________________________

If you had tested at the 5% significance level, would your conclusion have changed? Explain.

5. An engineer wanted to know whether the strength of one concrete mix (67-0-301) was less than that of another mixture (67-0-400). He randomly selected 9 mixtures of 67-0-301 and 12 mixtures of 67-0-400 and measured their respective strengths (lbs/in^2). Use the results below to test at the 5% significance level and you may assume normality.

Group	N	Mean	Std. Dev.	Std. Error
301	9	3668.889	458.52	152.84
400	12	4480.833	437.73	126.36

If Variances Are	t statistic	Df	P-value
Equal	-4.123	19	0.0003
Not Equal	-4.094	16.92	0.0004

95% Confidence Interval for the Difference between Two Means

Lower Limit	Upper limit
-1224.13	-399.76

X:____

H_o:____

H_a:____

Test Statistic:________________________________

Conclusion:__________________________________

____

____

____

____

P-Value:____________________________________

6. Researchers wanted to determine whether carpeted rooms contained more bacteria than uncarpeted rooms. To determine the amount of bacteria in a room, researcher pumped the air from the room over a Petri dish for a random selection of 8 carpeted rooms and 14 uncarpeted rooms. Use the results below to test at the 5% significance level and assume normality.

Group	N	Mean	Std. Dev.	Std. Error
Carpet	8	12.45	1.5344	0.5425
Uncarpet	14	7.31	3.8418	1.0268

If Variances Are	t statistic	Df	P-value
Equal	3.595	20	0.0009
Not Equal	4.429	18.58	0.0002

95% Confidence Interval for the Difference between Two Means

Lower Limit	Upper limit
2.16	8.13

X:____

H_o:____

H_a:____

Test Statistic:________________________________

Conclusion:__________________________________

____

____

____

____

P-Value:____________________________________

Identify the following data sets as discrete or continuous.

1. The number of engine revolutions per minute in automobiles.

2. The thickness of the polar ice cap in several locations.

3. The number of people hired by a company during certain weeks.

4. Your score on the Scholastic Assessent Test (SAT)

5. The amount of sand used on roads during winters in Houghton, MI.

Types of Cancer	Percentage
Breast	16.4%
Melanoma	14.6%
Lung	11.2%
Prostate	10.7%
Lymphoma	4.0%
Other	43.1%

Amount of Pizza Bill ($)	Number of Bills
0.00 – 5.00	1
5.00 – 10.00	31
10.00 – 15.00	19
15.00 – 20.00	8
20.00 – 25.00	2

Frequency Distributions

1. A wildlife biologist who is interested in the relationship between the length and the weight of a particular mammal at age 12 months collected a sample of 25 offspring of this species. The weight of the offspring (in pounds) are as follows:

Animal Number	Weight (lbs)
1	16.4
2	16.3
3	18.2
4	14.8
5	14.7
6	16.4
7	17.3
8	13.0
9	18.8
10	16.3
11	15.0
12	16.8
13	16.0
14	18.4
15	13.5
16	12.4
17	15.6
18	16.6
19	16.4
20	14.1
21	17.9
22	17.7
23	16.3
24	16.4
25	15.2

a. What is the variable of interest?

b. Is this variable quantitative or qualitative?

c. Is this variable continuous or discrete?

d. What is the observational unit?

e. Create a frequency distribution for these data.

Weight (lbs)	Frequency	Relative Frequency	Cumulative Frequency	Cumulative Relative Frequency

f. Draw a stem and leaf diagram for these data.

Questions Asked From Frequency Distributions

1. How many of the mammals weighed at least 13 but less than 14 pounds?

2. What proportion of the mammals weighed at least 16 but less than 17 pounds?

3. What proportion of the mammals weighed less than 15 pounds?

4. How many of the mammals weighed at least 16 pounds?

2. A study was conducted to determine the amount of annual rainfall (inches) in regions of Arizona. To conduct the study, a sample of water gauges was placed in various regions within the state to measure the amount of rainfall during the 12-month period from January 1999 through December 1999. The data was summarized in the following table:

Annual Rainfall (in)	Number of Regions
2.5 – 5.5	8
5.5 – 8.5	21
8.5 – 11.5	24
11.5 – 14.5	18
14.5 – 17.5	17
17.5 – 20.5	5
20.5 – 23.5	5
23.5 – 26.5	3
26.5 – 29.5	2
29.5 – 32.5	1

a. What is the variable of interest for this study?

b. What is the observational unit?

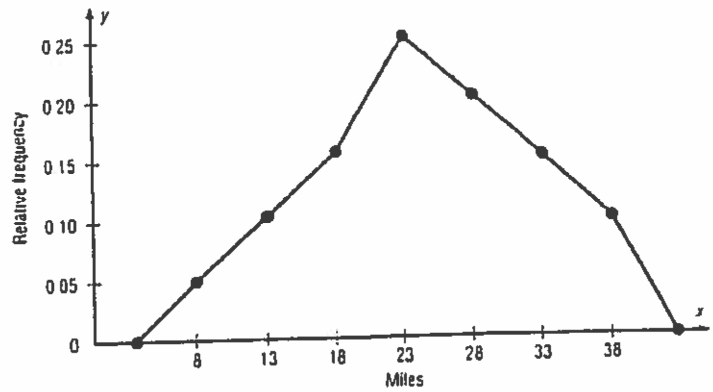
c. What is the population of interest?

d. How many regions were sampled?

e. How many regions reported an annual rainfall of less than 8.5 inches?

f. What proportion of the regions reported an annual rainfall of at least 26.5 inches and less than 29.5 inches?

3. Below is a graph of the miles that 20 randomly selected runners ran during a given week.



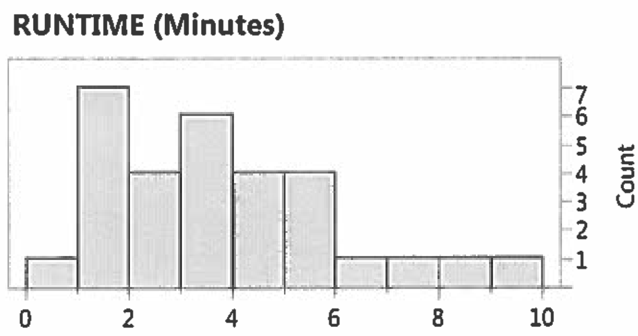
a. What type of graph is depicted here?

b. What is the width of the classes?

c. What type of distribution do these data display and explain what this means in terms of this particular situation?

d. What is the frequency for the first class?

4. Thirty students in an experimental psychology class use various techniques to train a rat to move through a maze. At the end of the course, each student's rat is timed as it negotiates the maze. The results (in minutes) are shown below.



a. What is the variable of interest?

b. Is this variable quantitative or qualitative?

c. What type of graph has been displayed?

d. Describe the type of distribution of these data and what this means for this particular situation.

e. How many rats took from 2 up to 4 minutes to negotiate the maze?

f. What proportion of rats took at least 1 minute to negotiate the maze?

Picturing Distributions of Data

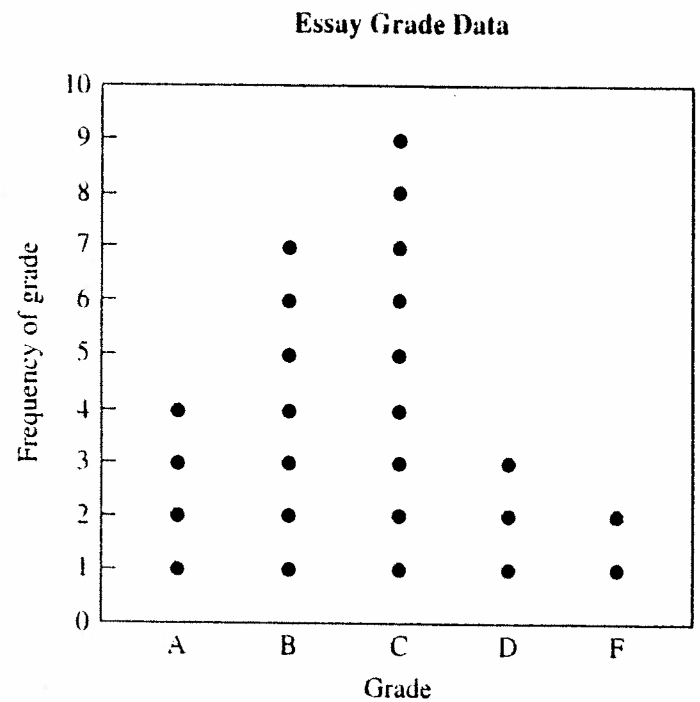
Qualitative Variables:

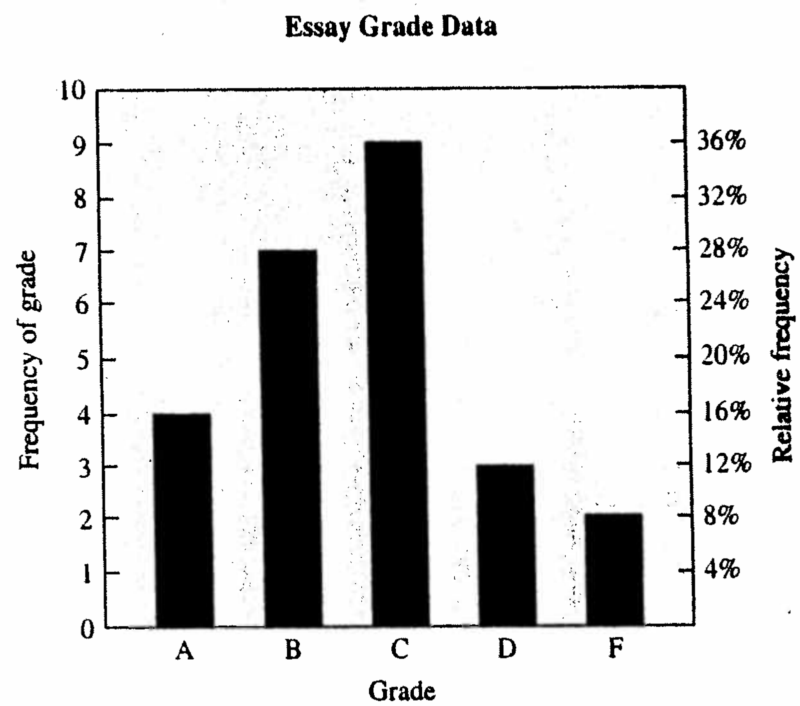
Bar Graph– consists of bars representing frequencies (or relative frequencies) for particular categories. The bar lengths are proportional to the frequencies.

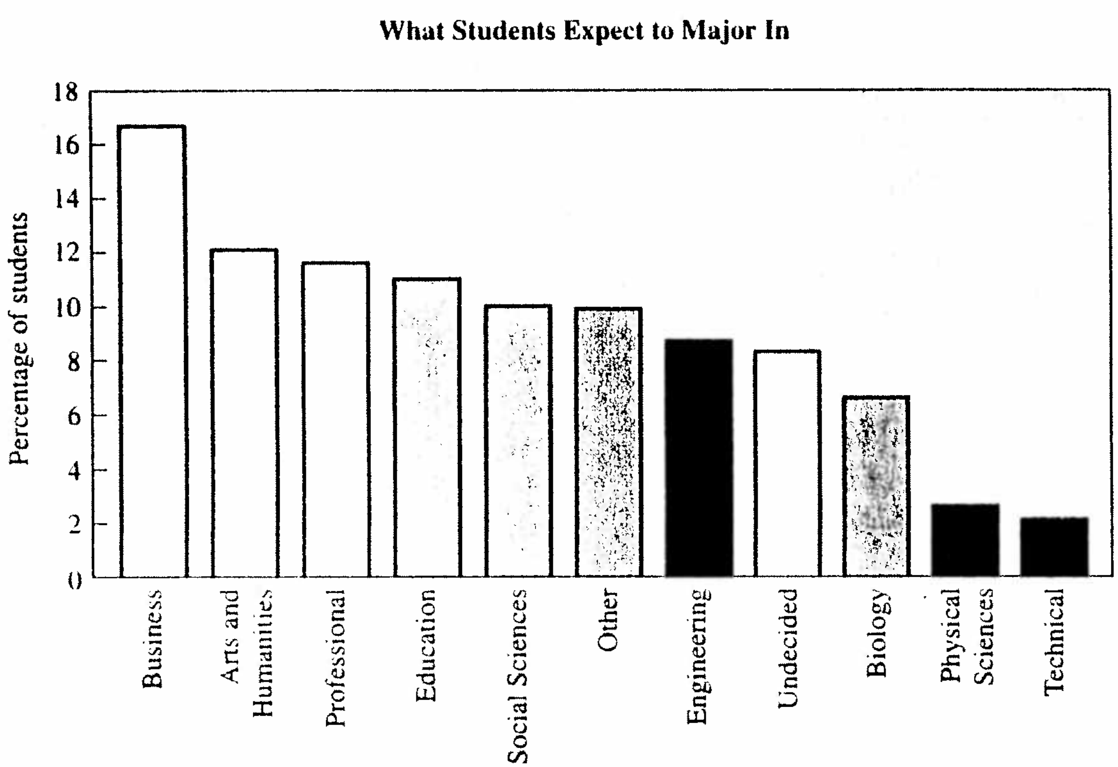
Dot Plot–similar to a bar graph, except each individual data value is represented with a dot.

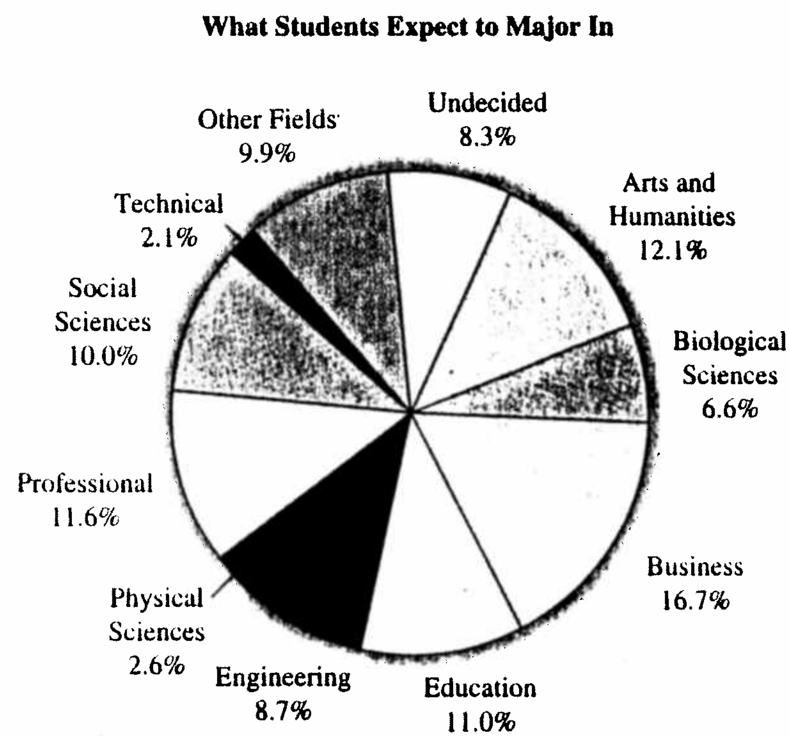
Pareto Chart–a bar graph with the bars arranged in frequency order.

Pie Chart–a circle divided so that each wedge represents the relative frequency of a particular category. The wedge size is proportional to the relative frequency.


Essay Grade Data
Frequency of grade
10
9
8
7
6
5
4
3
2
1
0
A
B
C
D
F
Grade


Essay Grade Data
Frequency of grade
10
9
8
7
6
5
4
3
2
1
0
A
B
C
D
F
Grade
36%
32%
28%
24%
20%
16%
12%
8%
4%
Relative frequency


What Students Expect to Major In
Percentage of students
18
16
14
12
10
8
6
4
2
0
Business
Arts and Humanities
Professional
Education
Social Sciences
Other
Engineering
Undecided
Biology
Physical Sciences
Technical


What Students Expect to Major In
Other Fields 9.9%
Undecided 8.3%
Technical 2.1%
Arts and Humanities 12.1%
Social Sciences 10.0%
Biological Sciences 6.6%
Professional 11.6%
Business 16.7%
Physical Sciences 2.6%
Engineering 8.7%
Education 11.0%

Histogram: displays data by using contiguous vertical bars whose heights correspond to the frequency or relative frequency of each class. You may use class boundaries to avoid gaps.

Frequency Polygon: displays data by using lines that connect points plotted for the frequencies or relative frequencies at the midpoint of each class. The height of each point corresponds to either frequency or relative frequency.

Ogive: displays the cumulative frequency or the cumulative relative frequency of data using the same methodology as the frequency polygon.

In all cases:

a. The x–axis represents the response variable.

b. The y–axis represents frequency or relative frequency for the histogram or frequency polygon and cumulative frequency or cumulative relative frequency for the ogive.

Single Numerical Measures

Numerical Measures of Central Tendency-where the data cluster, or center, about certain numerical values

Mean

Median

Mode

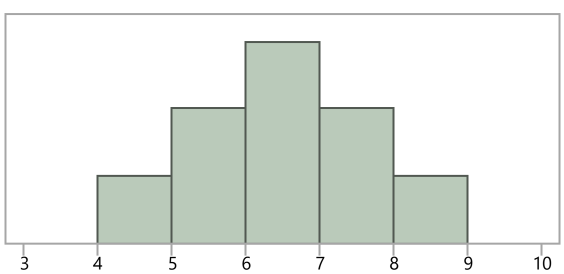
Numerical Measures of Variability- the spread of the data

Range

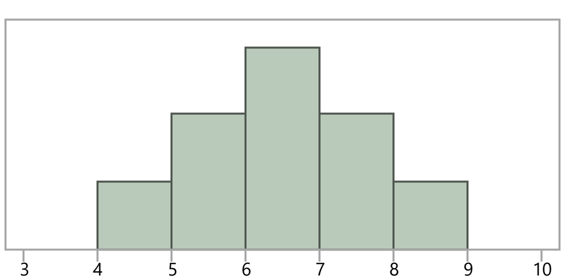
Variance

Standard Deviation

Interquartile Range



↑
CENTER



⟨← SPREAD →⟩

Measures of Central Tendency and Dispersion

Strength is an important characteristic of materials used in prefabricated housing. A random sample of eleven prefabricated plate elements from one manufacturer were subjected to a severe stress test and the maximum width (mm) of the resulting cracks were recorded.

0.68	0.59	0.92	0.48	3.52	3.13	1.49
1.28	1.03	2.54	2.65			

Calculate the following:

Mean

Median

Mode

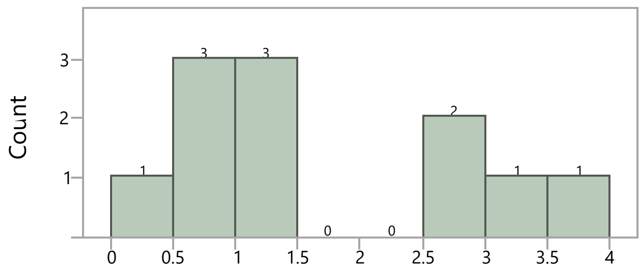
Variance

Standard Deviation

Range

Distributions

Width (mm)



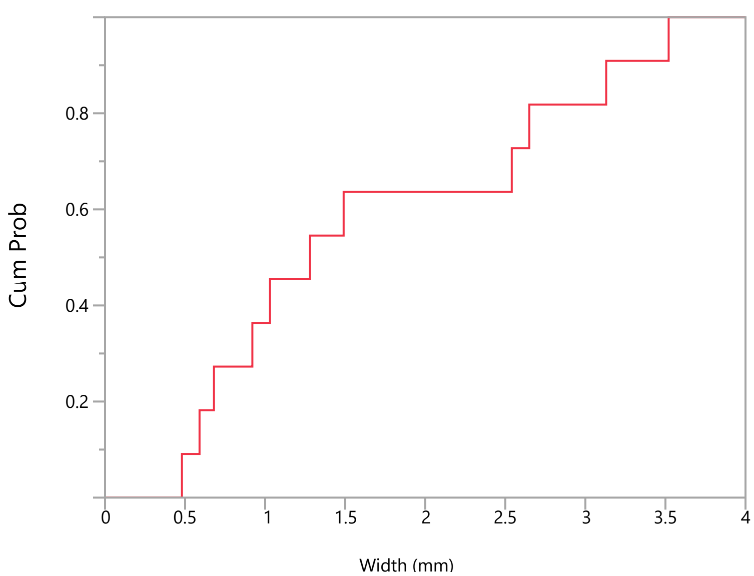
Quantiles

100.0%	maximum	3.52
99.5%		3.52
97.5%		3.52
90.0%		3.442
75.0%	quartile	2.65
50.0%	median	1.28
25.0%	quartile	0.68
10.0%		0.502
2.5%		0.48
0.5%		0.48
0.0%	minimum	0.48

Summary Statistics

Mean	1.6645455
Std Dev	1.095001
Std Err Mean	0.3301552
Upper 95% Mean	2.4001772
Lower 95% Mean	0.9289137
N	11

CDF Plot



We have 3 data sets all with $\bar{x}$ = 10

8, 8, 9, 10, 11, 12, 12

5, 6, 8, 10, 12, 14, 15

1, 2, 5, 10, 15, 18, 19

We have 3 other data sets that also all have $\bar{x}$ = 10

8, 9, 10, 10, 10, 11, 12

5, 7, 9, 10, 11, 13, 15

1, 5, 8, 10, 12, 15, 19

Understanding the Standard Deviation:

1. Allows comparison of variability between different data sets if you have the same units of measure.

2. It is an essential component of all inferential statistics.

3. Within a given data set, if the distribution is mound shape and somewhat symmetrical,

If we want to compare different data sets on a standardized basis, we can use z-scores which measure how many standard deviations any given observation is from its particular mean.

a. Z-scores can be used to compare observations from different samples with different units of measure.

b. Z-scores allow us to look at outliers.

Suppose $x_1, x_2, \ldots, x_n$ is a set of *n* observations with mean $\bar{x}$ and standard deviation *s*. The z-score corresponding to the i*th* observation, x_i, is given by

$$z_i = \frac{xi - \bar{x}}{s}$$

The owner of the Jungle Pet Store is trying to establish a policy for the return of animals. In a random sample of the lifetime (months) of pet guinea pigs, the sample mean lifetime was 60 months and the sample standard deviation was 15.5 months. One of the guinea pigs in this sample lived 48 months. Is this a reasonable lifetime, or should the store provide some sort of refund (or a new guinea pig)?

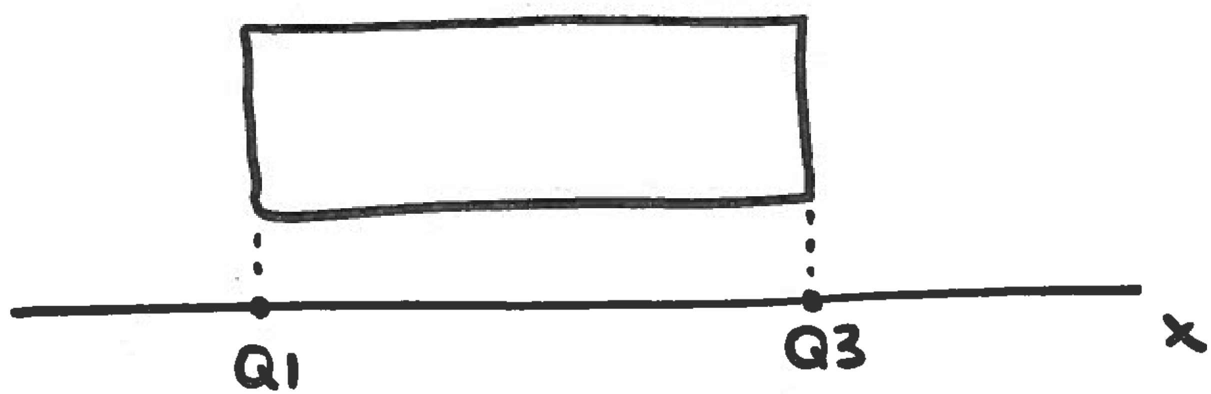
Most college career counselors agree that starting salary is associated with academic major. Even if a person's first job is not directly related to his or her course of study, the salary may still be related to the person's academic major. A recent survey of academic major and starting salary of graduates showed the following information.

Major	Mean	Standard Deviation
English	\$37,300	\$1175
Computer Science	\$52,500	\$2375

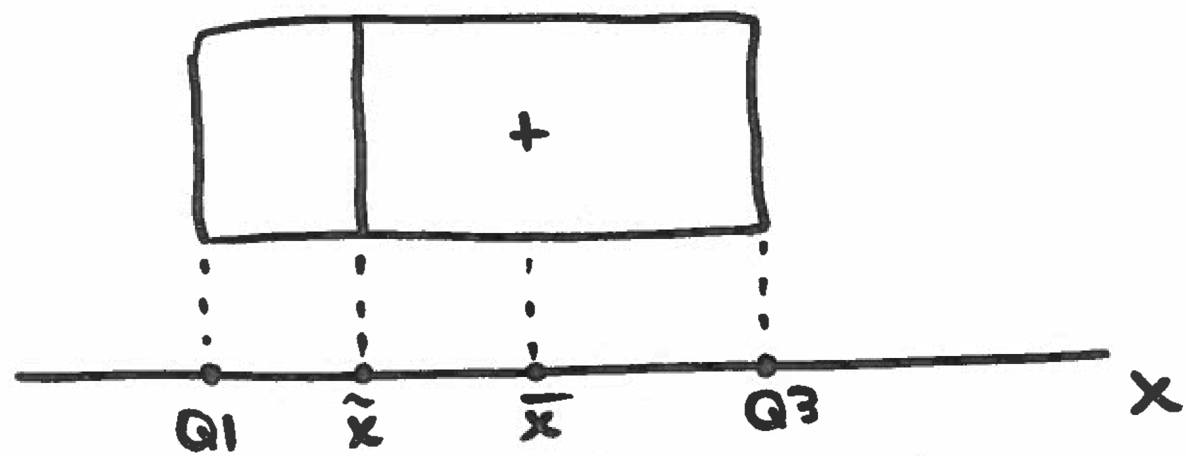
One Computer Science major who responded to the survey received a starting salary of \$55,000 and an English major received an offer of \$40,000. Which salary is *better*, in terms of statistics?

Boxplot is composed of a box and whiskers

1) The endpoints of the box are delineated by Q1 and Q3 (they are called the hinges).



2) A line bisecting the box is the median.
(Optional: the mean is delineated by + or *)



3) Lines drawn out from the box are called whiskers. To determine the endpoints of the whiskers, we first work with "inner fences".

First:

$$\text{step} = 1.5 * \text{IQR}$$

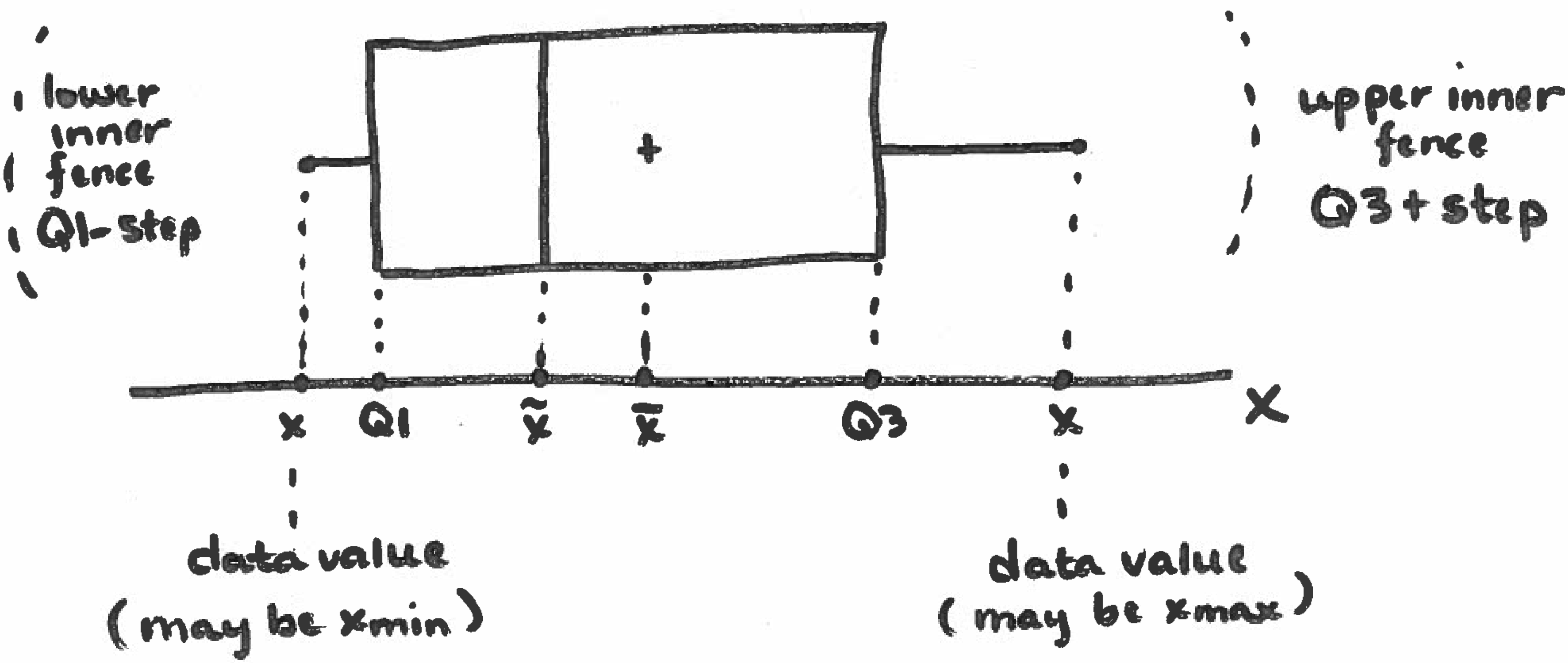
where IQR (interquartile range) = Q3 − Q1

"Inner Fences" determine what are the lowest and highest values of your variable that can be considered "good data".

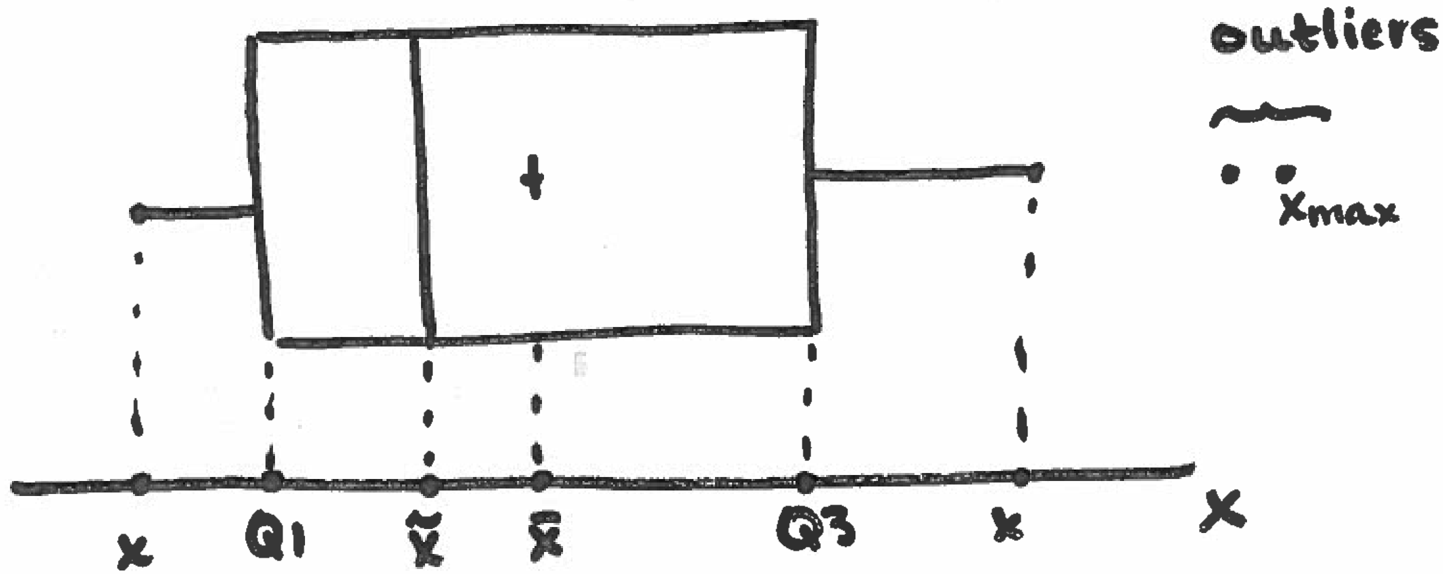
Q1 − step   lower fence value

Q3 + step   upper fence value

The whisker endpoints are the <u>data values</u> closest, but still within the inner fence values.



4) Any data values outside the whisker endpoints are considered outliers.



5) There are 2 classifications of outliers.

mild outliers - located between inner and outer fences

extreme outliers - located outside outer fences

where:

inner fences are

$Q1 - 1.5 * IQR$

$Q3 + 1.5 * IQR$

outer fences are

$Q1 - 3 * IQR$

$Q3 + 3 * IQR$

Quartiles and Boxplots

Eck Industries, Inc. manufactures cast aluminum cylinder heads that are used for liquid-cooled aircraft engines. The wall thicknesses of the coolant jackets are critical, particularly in high-altitude applications. Engineering specifications require that this thickness must be at least 0.120 in. The thicknesses (in inches) for a sample of 18 cylinder heads as measured by ultrasound, which is a nondestructive technique, are listed here:

0.223	0.193	0.218	0.201	0.231	0.204
0.228	0.223	0.215	0.223	0.237	0.226
0.214	0.213	0.233	0.224	0.217	0.210

From these data we get the following stem and leaf diagram.

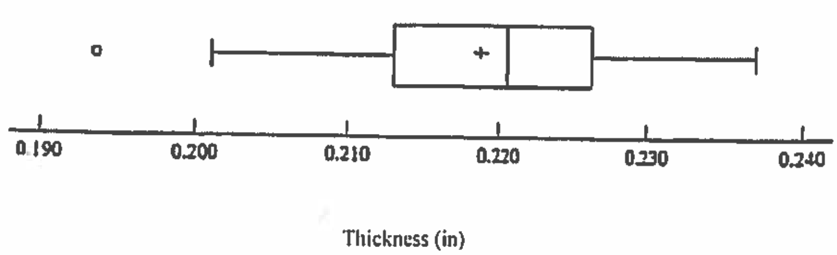
Stem	Leaves
0.19:	3
0.20:	1 4
0.21:	8 5 4 3 7 0
0.22:	3 8 3 3 6 4
0.23:	1 7 3

Key: 0.19: 3 = 0.193 inches

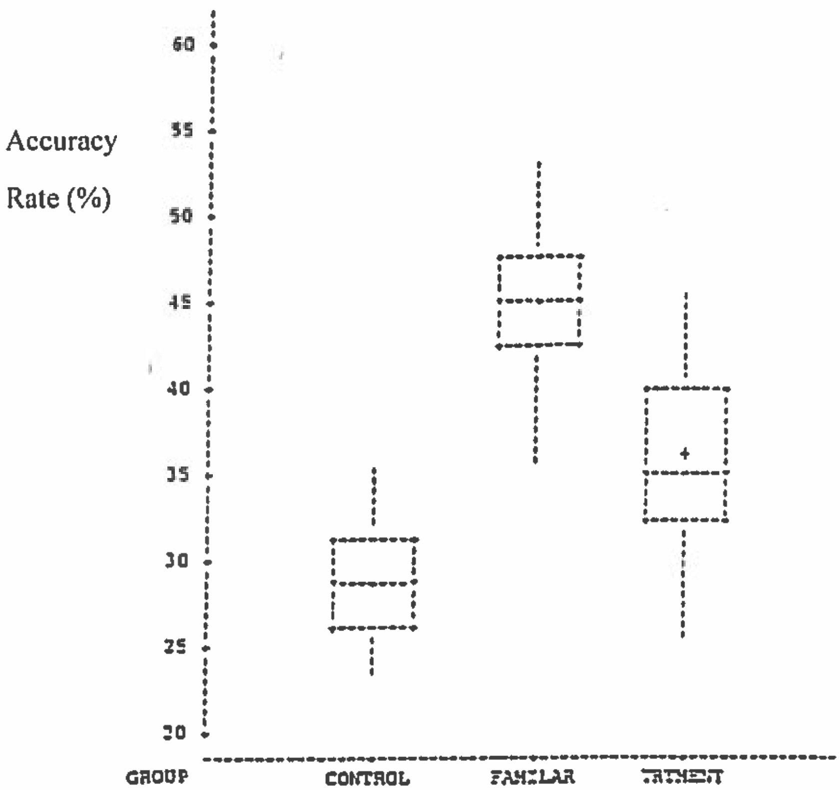
Calculate the mean, the median, and the upper and the lower quartiles.

Calculate 1.5*IQR to determine the endpoints of the inner fences and the endpoints of the whiskers.

We get the following modified boxplot. Are there any outliers?



In a study published in the *American Journal of Speech-Language Pathology* a sample of three groups of college students listened to an audio tape of a woman with imperfect speech. The groups were called control, treatment, and familiarity. At the end of the session, each student transcribed the spoken words. Boxplots were constructed for the percentage of words correctly transcribed (i.e., accuracy rate) by each group and are shown below.



a. What are the median accuracy rates for each of the three groups?

b. How does the variation of the accuracy rates compare for the three groups?

c. Do all three groups appear symmetrical? Why or why not?

Bayes's Rule

Given k mutually exclusive and exhaustive events, B_1, B_2, ..., B_k, such that $P(B_1) + P(B_2) + \ldots + P(B_k) = 1$, and an observed event A, then

$$P(B_i \mid A) = \frac{P(B_i \cap A)}{P(A)}$$

$$= \frac{P(B_i)\, P(A \mid B_i)}{P(B_1)\, P(A \mid B_1) + P(B_2)\, P(A \mid B_2) + \ldots + P(B_k)\, P(A \mid B_k)}$$

Conditional Probability and Independence

1. What is the probability that two cards dealt from a well-shuffled standard deck will both be queens where each card is not replaced after being drawn?

2. The probability that a newborn human baby is a boy is approximately 0.51. What is the probability that all three singleton babies born in a certain hospital in one day are boys? Given the first three singleton babies born were boys, what is the probability that the next singleton birth is a boy?

3. Tires salvaged from a train wreck are on sale at the GETRICH Tire Company. Of the 15 tires offered in the sale, 5 have suffered internal damage and the remaining 10 tires are damage free. If you were to randomly select 2 of these tires, what is the probability that

a. Both tires are damage free?

b. Exactly one of the tires is damage free?

c. At least one of the tires is damage free?

4. A manufacturer of hard drives knows that a shipment of 30 hard drives sent to a large discount store contains six that are defective. The manufacturer also knows that the store will randomly choose two of the hard drives, test them, and accept the shipment if neither is defective.

a. What is the probability that the first hard drive chosen by the store will be defective?

b. Given that the first hard drive passed inspection, what is the probability that the second chosen will fail inspection?

c. What is the probability that the shipment will be accepted?

5. A Baltimore detective is suspicious about 3 deaths that were determined by the department to be “accidental”. According to the data from the *Statistical Abstract for the United States*, there is a 0.0478 probability that a death is caused by an accident. If the 3 deaths under investigation are completely unrelated,

a. Find the probability that all 3 were accidental.

b. Find the probability that exactly 1 was accidental.

c. Find the probability that at least one was not accidental.

Probability

Simple: $P(A) = \frac{m}{N}$

Disjunctive: $P(A \cup B) = P(A \text{ or } B)$

$= P(A) + P(B) - P(A \text{ and } B)$

Joint: $P(A \cap B) = P(A \text{ and } B) = 0$

if A and B are mutually exclusive events

$P(A \cap B) = P(A \text{ and } B) = P(B) * P(A \mid B)$

$= P(A) * P(B \mid A)$

if A and B are dependent events

$P(A \cap B) = P(A \text{ and } B) = P(A) * P(B)$

if A and B are independent events

Conditional: $P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$, $P(B) \neq 0$

if A and B are dependent events

$= P(A)$

if A and B are independent events

$= 0$

if A and B are mutually exclusive events

ADDITION RULES FOR PROBABILITY

The number of reported cases of vaccine-preventable diseases in Canada in 2011 is given in the following table.

Disease	Frequency
Hib meningitis	38
Measles	759
Mumps	282
Pertussis	676
Other	8

Consider the events: A = {the case is measles}, B = {the case is mumps or pertussis}, C = {the case is not Hib meningitis}

Find the following probabilities.

a. $P(A)$

____________________________(2)

b. $P(A \cap C)$

____________________________(2)

c. $P(A \cup B)$

____________________________(2)

d. $P(A' \cap C)$

____________________________(2)

A store manager recorded the number of shoppers leaving his store in a one hour period. He observed the shopper's gender and whether a purchase was made or not.

	Purchase	No Purchase
Male	10	9
Female	37	13

a. Use a Venn diagram to illustrate the sample space.

b. What is the probability that a randomly selected shopper was female and made a purchase?

c. What is the probability that a randomly selected shopper was female?

d. What is the probability that a randomly selected shopper was male or that a purchase was made?

e. What is the probability that a randomly selected shopper was male or female?

A human resources director constructed the following two-way table to describe the probability that a managerial prospect will fall in a particular talent-motivation level.

		Talent	
Motivation	High	Medium	Low
High	0.05	0.16	0.05
Medium	0.19	0.32	0.05
Low	0.11	0.05	0.02

Suppose the director has decided to hire a new manager and defines the following events:

A: Prospect places in high motivation category

B: Prospect is medium or better in both categories

C: Prospect places low in at least one category

a. Draw a Venn diagram for this situation.

b. Find each of the probabilities for A, B, C.

Of all those people who enter Uncle's Stereo, a discount electronics store in New York City, 28% purchase a DVD player, 5% buy a large screen TV, and 4% buy both.

a. Draw a Venn diagram with probabilities to illustrate the relationship between two events, A: a person purchases a DVD player and B: a person buys a large screen TV.

b. Suppose a customer is selected at random. What is the probability that the customer buys a large screen TV or a DVD player?

c. Suppose a customer is selected at random. What is the probability that the customer buys only a DVD player?

d. Suppose a customer is selected at random. What is the probability that the customer buys only a large screen TV?

e. Suppose a customer is selected at random. What is the probability that the customer buys neither?

Marginal Probability and Baye’s Theorem

1. The probability that a recently transplanted shrub will survive is 0.80 if it rains tomorrow and 0.30 if it doesn’t rain tomorrow. If the probability of rain tomorrow is 0.1, what is the probability that the shrub will survive?

2. For people in a certain population, the probability of developing lung cancer is 0.12. If 40% of the population are smokers and 90% of those who develop lung cancer are smokers, what is the probability that a smoker in this population will develop lung cancer?

3. A company that manufactures shoes has 3 factories. Factory 1 produces 25% of the company's shoes, Factory 2 produces 60%, and Factory 3 produces 15%. One percent of the shoes produced by Factory 1 are mislabeled, 0.5% of those produced by Factory 2 are mislabeled, and 2% of those produced by Factory 3 are mislabeled.

a. If you purchase one pair of shoes made by this company, what is the probability that the shoes are mislabeled?

b. The shoes are mislabeled; what is the probability that they came from Factory 3?

4. Experience shows that the probability that a skin test will indicate the presence of an allergy to a particular substance when in fact the patient is not allergic to the substance under study is 0.10. The probability that the test will not detect the allergy in patients who actually are allergic to the substance is 0.15. Some 30 % of the persons tested are actually allergic to the substance. A patient is selected at random.

a. What is the false positive rate for this test?

b. What is the false negative rate for this test?

c. What is the probability that the patient is allergic to the substance given that the test came back positive indicating an allergy?

Bayes's Rule

Given k mutually exclusive and exhaustive events, $B_1, B_2, \ldots, B_k$, such that $P(B_1) + P(B_2) + \ldots + P(B_k)$, and an observed event A, then

$$P(B_i / A) = \frac{P(B_i \cap A)}{P(A)}$$

$$P(B_i / A) = \frac{P(B_i)\, P(A / B_i)}{P(B_1)\, P(A / B_1) + P(B_2)\, P(A / B_2) + \ldots + P(B_k)\, P(A / B_k)}$$

INTRODUCTION TO PROBABILITY

1. A fair coin is tossed and a heads or tails is observed. If a heads is observed, the coin is tossed a second time. If a tails results on the first toss, a fair die is then rolled. Use a list or a tree diagram to illustrate the sample space.

2. The Legend Bank in Bowie, Texas, has five tellers: 1 and 2 are trainees; 3, 4, and 5 are veterans. Tellers 2, 3, and 4 are female, and tellers 1 and 5 are male. At the end of the day, two tellers will be randomly selected and all of their transactions for the day will be audited.

    a. Use a list to construct the sample space for this experiment.

    b. What is the probability that both trainees will be selected for an audit?

    c. What is the probability that one male and one female will be selected for the audit?

Properties of a Binomial Random Variable

1. The experiment consists of n identical trials.

2. Each trial can result in only one of two possible (mutually exclusive) outcomes. One outcome is usually designated a success (S) and the other a failure (F).

3. The outcomes of the trials are independent.

We are interested in X = the number of successes in n trials where X = 0, 1, 2, …n

1. A Gallup Poll found that 78% of Americans worry about the quality and healthfulness of their diet. If four people are selected at random,

   a. Demonstrate that X, the number of people who worry about the quality and healthfulness of their diet, is a binomial random variable.

   b. Draw a tree diagram to illustrate the sample space.

Let's look at the probability that exactly 3 of the 4 people selected do worry about their diet.

There are 4 different sets of branches where you have 3 "successes" and 1 "failure".

The probability along any one of those branches is $(0.78)^3\ (0.22)$

So

$$P(X = 3) = 4\ (0.78)^3\ (0.22)$$

$$= (\text{Number of sample points for which } X = 3)\ p^{\text{number of successes}}\ q^{\text{number of failures}}$$

Or

Binomial Probability Distribution

$$P(X = x_o) = \binom{n}{x} p^x q^{n-x} \qquad \text{for } X = 0, 1, 2, \ldots, n$$

Where

p = probability of a success on a single trial

$q = 1 - p$

n = number of trials

x = number of successes in n trials

$\binom{n}{x} = \frac{n!}{x!(n-x)!}$

2. United Airline Flight 470 from Denver to St. Louis has an on-time performance of 60% (based on data collected from the reservation system). If 15 such flights over the next week are randomly chosen and their arrival time is noted.

Demonstrate that this is a binomial experiment.

Find the probability that among 15 such flights over the next week,

a. That exactly four arrive on time.

b. That fewer than six arrive on time.

c. That more than 13 arrive on time.

d. That between five and eight, inclusive, arrive on time.

e. That more than five but less than ten arrive on time.

f. That exactly seven do not arrive on time.

g. That at least six but less than nine do not arrive on time.

h. Find the mean number of flights that would arrive on time and the standard deviation for this distribution.

3. A survey found that one out of every six Americans say that he or she has visited a doctor in any given month. If nine people are selected at random,

   a. Demonstrate that this is a binomial experiment.

   b. Find the probability that exactly three will have visited a doctor last month.

   c. What is the mean number of Americans who visited the doctor last month?

   d. What is the standard deviation for this distribution?

4. The probability that a patient recovers from a delicate heart operation is 0.87. If seven patients who had this particular operation are randomly chosen,

   a. What is the probability that less than two will survive?

   b. On average, we would expect how many to survive?

   c. What is the standard deviation for this distribution?

   d. What is the probability that the number that survive is within 2 standard deviations of the mean?

5. Albuterol aerosol is manufactured by Warrick Pharmaceuticals, Ltd., and is used to treat and prevent wheezing, shortness of breath, and conditions caused by asthma. Based on a clinical trial, Warrick claims that only 10% of patients at least 12 years of age using this medication will experience tachycardia or a rapid heart rate, which is considered an adverse reaction. Suppose 30 people (at least 12 years old) who need this medication are selected at random. Each is given albuterol aerosol, and the number of people who experience tachycardia is recorded.

    a. Find the probability that at most 1 person will experience tachycardia.

    b. Suppose 7 people experience tachycardia. Is there any evidence to suggest that the company's claim is wrong? Justify your answer.

BINOMIAL DISTRIBUTION

TI 83 and TI 84

2nd DISTR (above the VARS key)

Arrow down to : binompdf OR binomcdf

Hit Return

binompdf(n, p, x) = $P(X = x_o)$ OR binomcdf(n, p, x) = $P(X \leq x_o)$

TI 89

CATALOG F3 B

Arrow down to : binompdf OR binomcdf

Hit Return

binompdf(n, p, x) = $P(X = x_o)$ OR binomcdf(n, p, x) = $P(X \leq x_o)$

binomcdf(n, p, x_1, x_2) = $P(x_1 \leq X \leq x_2)$

Discrete Random Variables

A student has not studied for a pop quiz. The quiz consists of three True/False question.

a. What is the probability of getting all three correct?

$P(C \text{ and } C \text{ and } C) = \frac{1}{8}$

b. What is the probability of getting only two correct?

$3\left(\frac{1}{8}\right) = \frac{3}{8}$

c. What is the probability of getting only one correct?

$3\left(\frac{1}{8}\right) = \frac{3}{8})$

d. What is the probability of getting none correct?

$P(C' \text{ and } C' \text{ and } C') = \frac{1}{8}$

Let X = the number of questions answered correctly. Fill in the table below with all possible values X may take on together with the respective probability associated with those values of X.

↓ # correct is a count so discrete

Number of correct answers (X)	$P(X = x_0)$
0	$\frac{1}{8}$
1	$\frac{3}{8}$
2	$\frac{3}{8}$
3	$\frac{1}{8}$
	1.0

Valid

1) $0 \le P(X = x_0) \le 1$ ✓

2) $\sum P(x_i) = 1$ ✓

Consider the following probability distribution:

X	-4	0	1	3
$P(X = x_0)$	0,1	0.2	0.4	0.3

a. Explain why this is or is not a valid probability distribution.

i. $0 \le P(X = x_0) \le 1$ ✓

ii. $\sum P(x_i) = 1$ ✓

b. Find $P(X = 0)$.

0.2

c. What value of X is most probable?

$X = 1$

d. What is the probability of $X = -2$?

$P(X = -2) = 0$

The Hard Rock Café in Dallas carefully monitors customer orders and has found that 70% of all customers ask for some kind of coffee, while the remainder order a specialized tea. Suppose three customers are selected at random. Let the random variable X be the number of customers who order coffee. Find the probability distribution for X.

customers is a count so discrete

Number of customers who order coffee: X	$P(X = x_0)$
0	$P(X=0) = (0.3)^3 = .027$
1	$P(X=1) = 3(.7)(.3)^2 = .189$
2	$P(X=2) = 3(.7)^2(.3) = .441$
3	$P(X=3) = (.7)^3 = .343$

1) $0 \le P(X = x_0) \le 1$ ✓

2) $\sum P(x_i) = 1$ ✓

"Kids Who Smoke": Looking at a particular school, all of the students were asked whether they smoke or not and the following information was retrieved.

age is a integer so discrete

Age: X	Number of "kids" who smoke	probability mass function $P(X = x_0)$	probability distribution function $P(X \le x_0)$
12	11	$P(X=12) = 11/650$	$11/650 = P(X \le 12)$
13	32	$32/650$	$43/650 = P(X \le 13)$
14	58	$58/650$	$101/650 = P(X \le 14)$
15	119	$119/650$	$220/650 = P(X \le 15)$
16	170	$170/650$	$390/650 = P(X \le 16)$
17	260	$260/650$	$650/650 \leftarrow P(X \le 17)$

$N = \sum f_i = 650$

$* \; P(X \le X_{max}) = 1$

If a "kid" is randomly chosen from this population, find the probability that

a. He/she is at most 15 years old.

$P(X \le 15) = P(X=12) + P(X=13) + P(X=14) + P(X=15)$

12, 13, 14, 15 $= 220/650$

b. He/she is at least 14 years old.

$P(X \ge 14) = P(X \le 17) - P(X \le 13)$

14, 15, 16, 17 $= 1 - 43/650$

$= 607/650$

c. He/she is at least 13 years old but less than 16 years old.

$P(13 \le X < 16) = P(X \le 15) - P(X \le 12)$

13, 14, 15 $= 220/650 - 11/650$

$= 209/650$

d. He/she is more than 13 years old but less than 16 years old.

$P(13 < X < 16) = P(X \le 15) - P(X \le 13)$

14, 15 $= 220/650 - 43/650$

$= 177/650$

e. Find the mean age of the smokers in this population and the standard deviation of this probability distribution.

$\mu = \sum_{i=1}^{6} x_i p(x_i) = (12)(11/650) + (13)(32/650) + \ldots + (17)(260/650)$

$\mu = 15.8$ years

$\sigma^2 = \sum_{i=1}^{6} x_i^2 p(x_i) - \mu^2 = \left[(12)^2(11/650) + (13)^2(32/650) + \ldots + (17)^2(260/650)\right] - (15.8)^2$

$\sigma^2 = 2.4$ years^2

$\sigma = \sqrt{2.4} = 1.5$ years

5. A patient complaining of severe stomach pains checked into a local hospital. After a series of tests, the doctors narrowed their diagnosis to four possible ailments. They believe there is a 40% chance that the patient has hepatitis; a 10% chance of cirrhosis; a 45% chance of gallstones; and a 5% chance of cancer of the pancreas. The doctors are certain that the patient has only one of the diseases, but will not know which disease until further tests are performed. The cost associated with treating each disease is given in the table below.

Disease	Hepatitis	Cirrhosis	Gallstones	Pancreatic Cancer
Cost	$700	$1,100	$3,320	$16,450

Construct the probability distribution for the cost of treating the patient and calculate the expected or average cost of treatment for a patient with these types of symptoms.

discrete r.v. $\Rightarrow$ valid ??

X (Cost of treatment) ($)	$P(X = x_0)$
700	0.4
1100	0.1
3320	0.45
16450	0.05

$\mu = E(X) = \sum_{i=1}^{4} x_i p(x_i)$

$= (700)(0.4) + (1100)(0.1)$

$+ (3320)(0.45)$

$+ (16450)(0.05)$

$\mu = \$2706.5$

Expected or average cost: $\mu = \$2706.5$

6. The exposure time for a tanning session depends on each individual model of sunlamp and the specific equipment, as well as skin type. Most sessions range from 10 to 30 minutes. Suppose the duration of a tanning session (in minutes) at the Paradise Island Day Spa in Augusta, Georgia, is a random variable with probability distribution given in the table below.

X (min)	10	12	15	20	25	30
$P(X = x_o)$	0.30	0.25	0.15	0.12	0.10	0.08

a. Find the mean duration of a tanning session.

$$\mu = \sum_{i=1}^{6} x_i p(x_i) = (10)(0.30) + (12)(0.25) + \ldots + (30)(0.08)$$

$$\mu = 15.55 \text{ or } 15.6 \text{ min}$$

b. Find the standard deviation for this probability distribution.

$$\sigma = \sqrt{\sum_{i=1}^{6} x_i^2 p(x_i) - \mu^2}$$

$$\sigma = \sqrt{\left[(10)^2(0.30) + (12)^2(0.25) + \ldots + (30)^2(0.08)\right] - (15.55)^2}$$

$$\sigma = \sqrt{282.55 - (15.55)^2} = 6.36 \text{ or } 6.4 \text{ min}$$

$\sigma = 6.2$ if use $\mu = 15.6$

c. Find the probability that a randomly selected session has a duration within two standard deviations of the mean.

$$P(\mu - 2\sigma < X < \mu + 2\sigma)$$

$$= P(15.55 - 2(6.36) < X < 15.55 + 2(6.36))$$

$$= P(2.8 < X < 28.3)$$

$$= P(X = 10) + P(X = 12) + P(X = 15) + P(X = 20) + P(X = 25)$$

$$= 1 - P(X = 30)$$

$$= 0.92$$

Discrete Random Variables

1. A student has not studied for a pop quiz. The quiz consists of three True/False questions.

a. Draw a tree diagram illustrating the sample space.

b. What is the probability of getting all three correct?

c. What is the probability of getting only two correct?

d. What is the probability of getting only one correct?

e. What is the probability of getting none correct?

f. Let X = the number of questions answered correctly. Fill in the table below with all possible values X may take on together with the respective probability associated with those values of X.

X (Number of correct answers)	$P(X = x_o)$

g. Graph this probability distribution.

2. Consider the following probability distribution:

X	-4	0	1	3
$P(X = x_o)$	0.1	0.2	0.4	0.3

a. Explain why this is or is not a valid probability distribution.

1.
2.

b. Find $P(X = 0)$.

c. What value of X is most probable?

d. What is the probability of $X = -2$?

3. The Hard Rock Café in Dallas carefully monitors customer orders and has found that 70% of all customers ask for some kind of coffee, while the remainder order a specialized tea. Suppose three customers are selected at random. Let the random variable X be the number of customers who order coffee. Find the probability distribution for X.

X (Number who order coffee)	$P(X = x_o)$

4. “Kids Who Smoke”: Looking at a particular school, all of the students were asked whether they smoke or not and the following information was retrieved.

X (Age)	Number of “kids” who smoked	$P(X = x_o)$	$P(X \leq x_o)$
12	11		
13	32		
14	58		
15	119		
16	170		
17	260		

If a “kid” is randomly chosen from this population, find the probability that

a. He/she is at most 15 years old.

b. He/she is at least 14 years old.

c. He/she is at least 13 years old but less than 16 years old.

d. He/she is more than 13 years old but less than 16 years old.

e. Find the mean age of the smokers in this population and the standard deviation of this probability distribution.

5. A patient complaining of severe stomach pains checked into a local hospital. After a series of tests, the doctors narrowed their diagnosis to four possible ailments. They believe there is a 40% chance that the patient has hepatitis; a 10% chance of cirrhosis; a 45% chance of gallstones; and a 5% chance of cancer of the pancreas. The doctors are certain that the patient has only one of the diseases, but will not know which disease until further tests are performed. The cost associated with treating each disease is given in the table below.

Disease	Hepatitis	Cirrhosis	Gallstones	Pancreatic Cancer
Cost	\$700	\$1,100	\$3,320	\$16,450

Construct the probability distribution for the cost of treating the patient and calculate the expected or average cost of treatment for a patient with these types of symptoms.

X (Cost of treatment)	$P(X = x_o)$

Expected or average cost:________________________________

6. The exposure time for a tanning session depends on each individual model of sunlamp and the specific equipment, as well as skin type. Most sessions range from 10 to 30 minutes. Suppose the duration of a tanning session (in minutes) at the Paradise Island Day Spa in Augusta, Georgia, is a random variable with probability distribution given in the table below.

X	10	12	15	20	25	30
$P(X = x_o)$	0.30	0.25	0.15	0.12	0.10	0.08

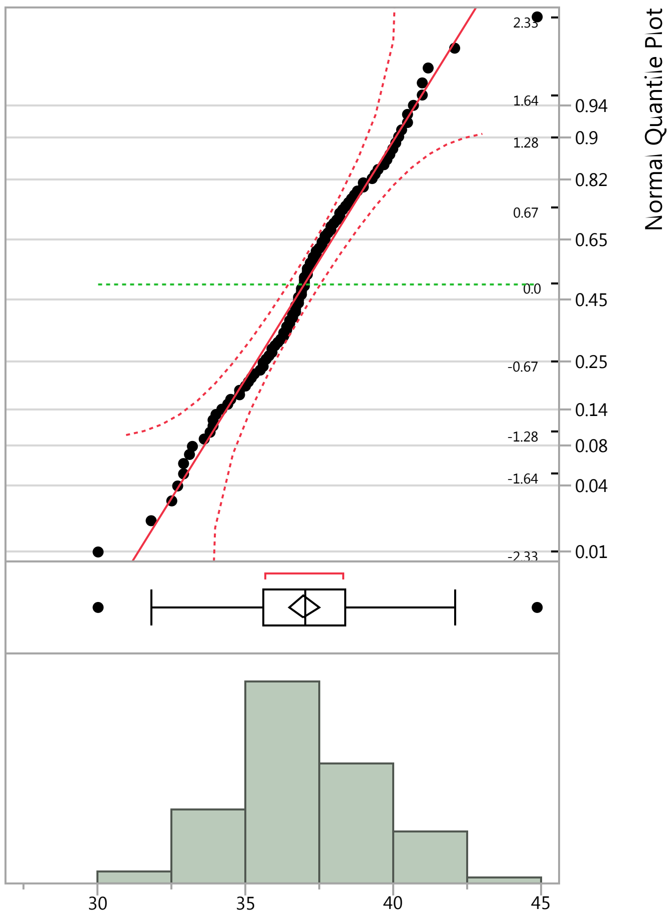
a. Find the mean duration of a tanning session.

b. Find the standard deviation for this probability distribution.

c. Find the probability that a randomly selected session has a duration within two standard deviations of the mean.

Distributions

Gas Mileage



Quantiles

100.0%	maximum	44.9
99.5%		44.9
97.5%		41.6275
90.0%		40.19
75.0%	quartile	38.375
50.0%	median	37
25.0%	quartile	35.625
10.0%		33.81
2.5%		32.1675
0.5%		30
0.0%	minimum	30

Summary Statistics

Mean	36.994
Std Dev	2.4178971
Std Err Mean	0.2417897
Upper 95% Mean	37.473763
Lower 95% Mean	36.514237
N	100

Stem and Leaf

Stem	Leaf	Count
44	9	1
43		
42	1	1
41	0 0 2	3
40	0 1 2 3 5 5 7	7
39	0 0 3 4 5 7 8 9	8
38	0 1 2 2 3 4 5 6 7 8	10
37	0 0 0 0 1 1 1 2 2 3 3 4 4 5 6 6 7 7 8 9 9	21
36	0 1 2 3 3 4 4 5 5 6 6 7 7 7 8 8 8 9 9 9	20
35	0 1 2 3 5 6 6 7 8 9 9	11
34	0 2 4 5 8 8	6
33	1 2 6 8 9 9	6
32	5 7 9 9	4
31	8	1
30	0	1

30|0 represents 30.0

Assessing Normality

The coronary care unit at Bayonet Point Hospital recently investigated the length of stay (days) of their patients. They took a random sample of patients and their length of stay is reported below.

5.5	7.5	3.0	5.0	4.0	4.5
6.0	5.0	6.5	3.5	4.5	4.5
6.5	3.5	4.0	5.0		

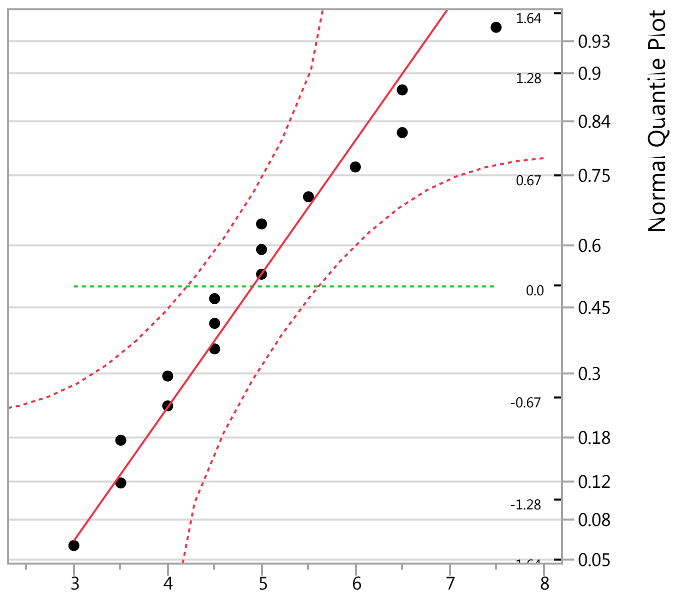
a. Draw a stem and leaf diagram of the sample data.

b. Calculate the descriptive statistics: mean, median, standard deviation, upper and lower quartiles.

c. Calculate the PI.

d. Calculate the IQR/s.

e. Examine the probability plot below.



f. Do these data suggest that they follow a normal distribution? Go through the stepwise procedure to justify your decision.

NORMAL DISTRIBUTION

To find probabilities for a given X value:

TI 83 and TI 84

2nd DISTR (above the VARS key)

Arrow down to : normalcdf

Hit return

normalcdf(lower, upper, mean, standard deviation)

For Infinity use 1EE99 or -1EE99 (located above the comma key)

TI 89

CATALOG F3 N

Arrow down to normcdf

Hit return

Normcdf(lower, upper, mean, standard deviation)

Infinity is located above the CATALOG key

To find X values for a given probability:

Use invnorm(cumulative probability, mean, standard deviation)

Normal Distribution

1. Let the random variable Z have a standard normal distribution.

   a. Find $P(-1.33 > Z)$

   b. Find $P(Z > 2.59)$

   c. Find $P(-1.56 < Z < -0.56)$

   d. Find $P(Z \leq -2.00 \cup Z \geq 2.00)$

2. Let the random variable Z have a standard normal distribution. Solve each expression for b.

a. $P(Z \leq b) = 0.8686$

b. $P(b < Z) = 0.0192$

c. $P(-b \leq Z \leq b) = 0.5527$

3. The physical fitness of a patient is often measured by the patient's maximum oxygen uptake (recorded in ml/kg). The mean maximum oxygen uptake for cardiac patients who regularly participate in sports or exercise programs was found to be 24.1 with a standard deviation of 6.30. Assume this distribution is approximately normal.

   a. What is the probability that a cardiac patient who regularly participates in sports has a maximum oxygen uptake of at least 20 ml/kg?

   b. Consider a cardiac patient with a maximum oxygen uptake of 10.5 or less. Is it likely that this patient participates regularly in sports or exercise programs? Explain.

4. The ability of unsaturated field soils to retain water is critical to the soil ecosystem. A team of soil scientists investigated the water retention properties of soil cores sample from an uncropped field consisting of silt loam. At a pressure of 0.1 megapascal (MPa), the water content of the soil (measured in m^3 of water/ m^3 of soil) was determined to be approximately normally distributed with a mean of 0.27 and a standard deviation of 0.04. For a soil core at a pressure of 0.1 MPa, what is the probability its water content is less than 0.30 m^3/m^3?

5. A group of Florida State University psychologists examined the effects of alcohol on the reactions of people to a threat. After obtaining a blood alcohol level (BAL) of at least 0.015, experimental subjects were placed in a room and threatened with electric shocks. Using sophisticated equipment to monitor the subjects' eye movements, the startle response (measured in milliseconds) was recorded for each subject. The mean and standard deviation of the startle responses were 37.9 and 12.4, respectively. Assume that the startle response, X, for a person with a BAL of at least 0.015 is approximately normally distributed.

   a. Find the probability that the startle response is between 40 and 50 milliseconds.

   b. Within what limits would you expect the middle 95% of startle response to fall?

6. A physical fitness association is including the mile run in their secondary school fitness test for boys. The time for this event for boys in this age group is approximately normally distributed with a mean of 450 seconds and a standard deviation of 40 seconds. If the association wants to give the fastest 10% a gold medal, what time should the association set for this criterion?

7. *Ecological Applications* (May 1995) published a study on the development of forest following wildfires in the Pacific Northwest. One variable of interest to the researcher was tree diameter at breast height (dbh) 100 years after the fire. The population of Douglas fir trees was shown to have an approximately normal diameter distribution, with a mean of 50 cm and a standard deviation of 12 cm. Find the diameter, d, such that 30% of the Douglas fir trees in the population have diameters that exceed d.

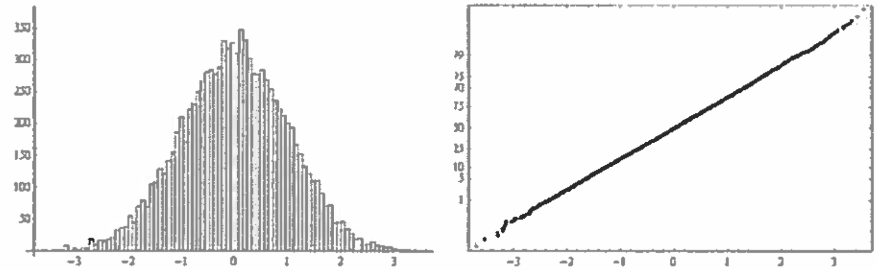
8. Dining room chairs come in many different woods, styles, and shapes. The height of the seat of a randomly selected oak dining room chair is approximately normal with mean 85 cm and standard deviation 1.88 cm.

   a. Find a value *h* such that 99% of all dining room chairs have a height less than *h*.

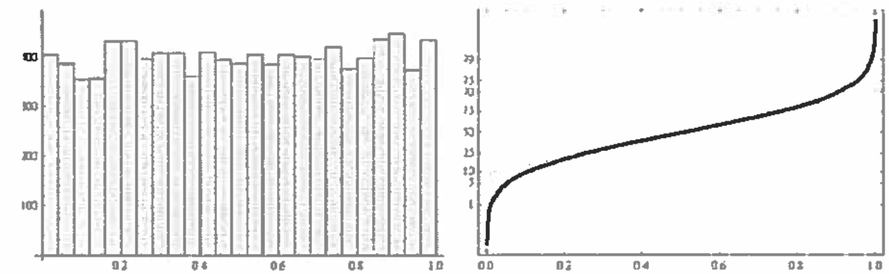
   b. There is some evidence to suggest that, after five years of use, the mean height of these chairs has decreased, due to wear, erosion, and humidity. Suppose that after five years, the probability that the height is more than 86 cm is 0.0718. Find the mean height after five years.

Histograms and QQ Plots of random samples of size $n = 10,000$ from various distributions

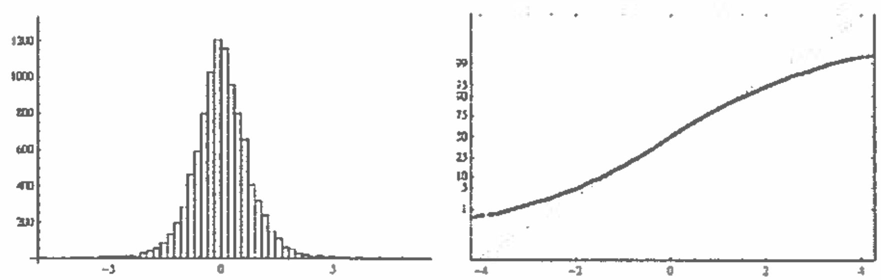
$Z \sim N(0,1)$



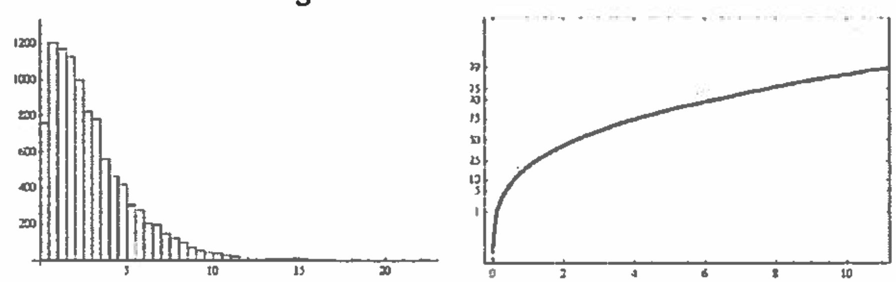
$UNIF(0,1)$ – weak tailed



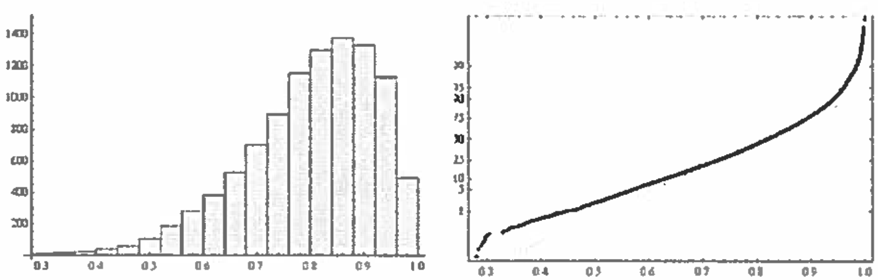
T_5 – heavy tailed



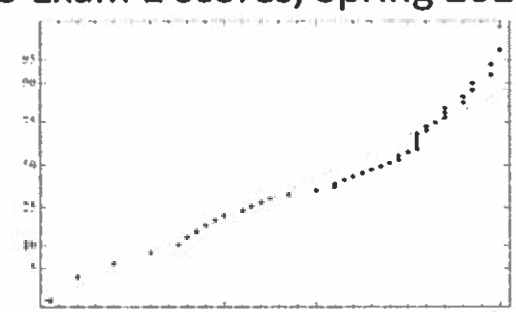
$x^2{}_3$ – right skewed



$BETA(\alpha = 8, \beta = 2)$ – left skewed



MA 3715 Exam 1 scores, Spring 2013 (n = 46)



Sampling Distribution for the Sample Mean, $\bar{x}$

Suppose we have a population of size N = 5 consisting of the ages of five children which are as follows:

$x_1 = 6$ $x_2 = 8$ $x_3 = 10$ $x_4 = 12$ $x_5 = 14$

Then the distribution of the data is as follows:

f

1

6 8 10 12 14

Ages

The population mean, μ is:

The population variance, σ^2 is:

Let's draw ALL possible samples of size n = 2 with replacement.

		Second Draw				
		6	8	10	12	14
First	6	(6,6)	(6,8)	(6,10)	(6,12)	(6,14)
Draw	8	(8,6)	(8,8)	(8,10)	(8,12)	(8,14)
	10	(10,6)	(10,8)	(10,10)	(10,12)	(10,14)
	12	(12,6)	(12,8)	(12,10)	(12,12)	(12,14)
	14	(14,6)	(14,8)	(14,10)	(14,12)	(14,14)

Construct the probability distribution for the sample mean, $\bar{x}$, where n=2.

$\bar{x}$	f	$P(\bar{x} = x_o)$

Now the graph of the sampling distribution for $\bar{x}$ is:

$P(\bar{x} = x_o)$

6 7 8 9 10 11 12 13 14

$\bar{x}$

Mean of the sampling distribution

$\mu_{\bar{x}}$

Variance of the sampling distribution

$\sigma^2_{\bar{x}}$

Confidence Intervals and Hypothesis Testing

1. Everyone who owns or has access to a computer probably has several passwords for various programs. Some psychologists believe passwords are predictable based on personality traits, and many computer hackers claim that any password can be *acquired*. In order to study computer security at Spectaguard Acquisition, a special program was developed and used to systematically try various passwords for randomly selected user accounts. A random sample of time (in hours) needed to obtain each password is given below. Find the 94% confidence interval for the true mean time.

1.88	1.71	2.09	6.60	2.28	3.52	2.64
4.94	1.78	4.13	2.55	1.66	0.28	4.02
4.47	0.68	5.67	0.03	1.89	1.76	2.10
6.58	2.30	3.50	2.66	4.92	1.80	4.11
2.57	1.64	0.26	4.00	4.49	0.66	5.69

$n = 35$ $\bar{x} = 2.910$ $s = 1.773$ Population: all Spectaguard Acquisition users

$\bar{x} \pm Z_{\alpha/2} * s/\sqrt{n}$

$2.910 \pm (1.881)\left(\frac{1.773}{\sqrt{35}}\right)$

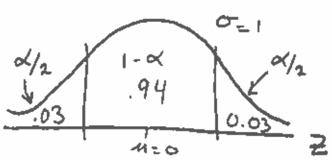
$= 2.910 \pm 0.564$

X: X = time to get password (hrs)

Assumptions: SRS

s is a good estimate of σ

$\bar{X}$ is normal because $n \geq 30$



invnorm$(0.03, 0, 1) = \pm 1.881$

Reliability Coefficient: $z = \pm 1.881$

(2.346 hrs, 3.474 hrs)

If the assumption is, on average, that it takes 3.5 hours to obtain each password, test this assumption using the confidence interval computed above. Explain why you reached your conclusion.

H_0 $\mu = 3.5$ hrs

H_a $\mu \neq 3.5$ hrs

Conclusion Reject H_0, 94% sure the mean time to obtain a password at Spectaguard Acquisition is different from 3.5 hours.

Why? 3.5 falls outside CI

2. Most major league baseball parks have a device to measure the speed of every pitch. The results are often displayed on a scoreboard and tracked by coaches. A random sample of pitches made during the first inning of baseball games around the country was selected. The speed of each pitch (in mph) is given below. Test at the 5% significance level to see if the mean speed is faster than the previous season mean of 84.25 mph. Assume the data is normally distributed.

85	90	82	86	83	88	87	90	92	84	92	90	89
90	89	84	87	92	89	86	89					

$n=21$ $\quad \bar{x} = 87.8$ $\quad s = 2.99$

Population: pitches in the first inning of baseball games this season

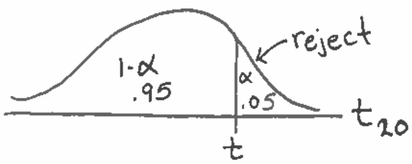
$$t_{obs} = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{87.8-84.25}{2.99/\sqrt{21}} = 5.44$$

X: X = Speed of pitch (mph)

Assumptions: SRS

s is a good estimate of σ

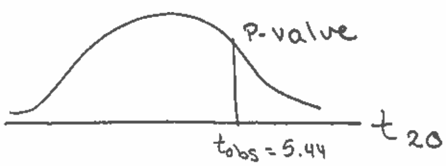
$\bar{x}$ is normally distributed because pitch speeds are normal



invt(.95,20) = 1.725

H_0 $\mu = 84.25$ mph

H_a $\mu > 84.25$ mph



tcdf(5.44, ∞, 20) = 0.000013

Test Statistic $t_{obs} = 5.44$

Rejection Region Reject H_0 if $t_{obs} \geq 1.725$

Conclusion Reject H_0, 95% sure the mean pitch speed for this season is faster than 84.25 mph

P-Value 0.000013

3. The mean income per year (in dollars) of employees who produce internal and external newsletters and magazines for corporation was reported to be \$51,500 in 2006. These editors and designers work on corporate publications, but not on marketing materials. Due to poor economic conditions and over-supply over the past two years, these corporate communications workers may be experiencing a decrease in salary. A random sample of 38 corporate communications workers revealed a sample mean of \$49, 762 with a sample standard deviation of \$3749. Test at the 5% significance level to determine whether there is any evidence to suggest that the mean income per year of corporate communications workers has decreased.

$n = 38$ $\qquad \alpha = 0.05$ $\qquad$ Population: all corporate communications workers

$\bar{x} = \$49,762$

$S = \$3,749$

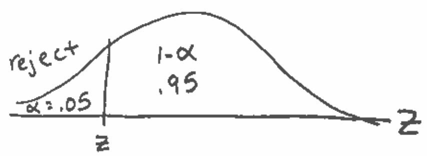
$$Z_{obs} = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{49,762 - 51,500}{3749/\sqrt{38}} = -2.858$$

X: X = income/year (\$)

Assumptions: SRS

S is a good estimate of σ

$\bar{x}$ is normally distributed because $n \geq 30$

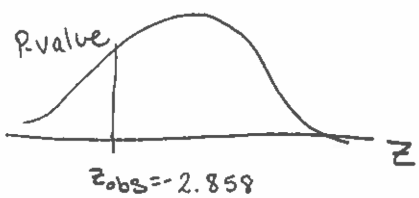


invnorm(.05,0,1) = -1.645

H_0 $\mu = \$51,500$

H_a $\mu < \$51,500$

Test Statistic $Z_{obs} = -2.858$



Rejection Region Reject H_0 if $Z_{obs} \leq -1.645$

Conclusion Reject Ho, 95% sure the mean income/year of corporate communications workers is less than \$51,500.

normcdf(-∞, -2.858, 0, 1) = 0.0021

P-Value 0.0021

Would your conclusion have changed if you had tested at the 1% significance level? Explain.

No, still reject Ho because p-value (0.0021) is less than α (0.01).

4. Most patients in need of a kidney transplant are placed on a dialysis machine. There is some evidence to suggest that the longer a patient remains on dialysis, the worse they fare following a transplant. A random sample of kidney transplant patients was obtained, and the wait time (in months) was recorded.

37.3	38.1	23.6	46.2	32.7	19.1	44.1	22.2	30.6	49.1	48.3	33.1	40.0
24.6	19.4	43.7	50.1	33.7	36.5	39.4	34.7	48.0	42.3	23.5		

Assuming normality, construct a 99% confidence interval for the mean wait times.

$n=24$ $\bar{x}=35.85$ $s=9.81$ P.e. ± R.C. (S.E) Population: all kidney transplant patients

$\bar{x} \pm t_{n-1,\alpha/2} * s/\sqrt{n}$

$35.85 \pm 2.807 * \frac{9.81}{\sqrt{24}}$

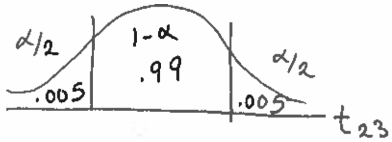
35.85 ± 5.621

X: X = wait time (months)

Assumptions: SRS

S is a good estimate of σ

$\bar{x}$ is normally distributed because wait times are normal.



invt (0.005, 23) = ± 2.807

Reliability Coefficient: $t = \pm 2.807$

(30.23 months, 41.47 months)

Use the computed confidence interval to see if the data suggests that the mean waiting time for a kidney transplant has changed from the accepted mean time of 45 months. Test at the 1% significance level.

H_0 $\mu = 45$ months

H_a $\mu \neq 45$ months

Conclusion Reject Ho, 99% sure the mean waiting time for kidney transplant patients is different from 45 months.

Why? 45 falls outside CI

5. According to the document *Consumer Expenditures*, a publication of the U.S. Bureau of Labor Statistics, the average consumer unit spent \$2749 on apparel and services in 2010. This year, 32 consumer units in the Northeast had the following annual expenditures, in dollars, on apparel and services.

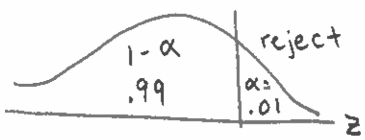
2292	2470	3033	2699	2286	3236	3246	3205	2624	2747	3701	3042
3212	2873	3457	2857	2778	3280	2535	3025	3003	2764	3215	2725
2808	2658	3005	3110	2990	2656	2558	2798				

Test at the 1% significance level to determine if this year's consumer units are spending more on apparel and services compared to that in 2010.

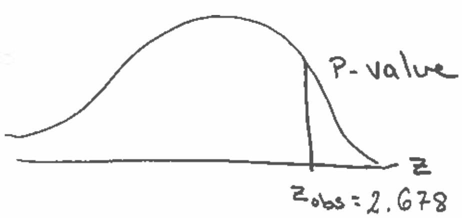
Population: all consumer units in the Northeast this year

$n=32 \quad \bar{x}=2902.75 \quad s=324.82$

$$Z_{obs} = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{2902.75-2749}{\frac{324.82}{\sqrt{32}}} = 2.678$$



invnorm(.99, 0, 1) = 2.33



normcdf(2.678, ∞, 0, 1) = 0.0037

X: X = annual expenditures (\$)

Assumptions: SRS

s is a good estimate of σ

$\bar{x}$ is normally distributed because $n \geq 30$

H_0 $\mu = \$2,749$

H_a $\mu > \$2,749$

Test Statistic $Z_{obs} = 2.678$

Rejection Region Reject H_0 if $Z_{obs} \geq 2.33$

Conclusion Reject H_0, 99% sure the mean annual expenditures for apparel and services with this year's consumer units is more than \$2,749.

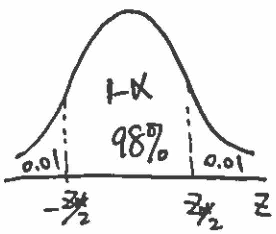
P-Value 0.0037

Based on your conclusion, what type of error have you subjected yourself to and explain what that error entails based on this specific situation.

Type I error - proved this year's consumer units spend more, on average, on apparel and services than in 2010, but in fact, they really didn't.

Optimal Sample Size

1. Suppose you wish to estimate the true mean weight of the liver of full grown frogs of a certain variety correct to within plus or minus 0.1 gm. Each frog costs \$6 and there is \$300 available for purchase. If frog liver weights are normally distributed and a population standard deviation of 0.35 gm is assumed and you want to be 98% confident, is the amount allotted sufficient?



X = weight of the frog liver (gm)

$B = \pm 0.1$ (gm)

$\sigma = 0.35$ (gm)

$100(1-\alpha)\% = 98\%$

$$n = \frac{Z_{\alpha/2}^2 * \sigma^2}{B^2} = \frac{(-2.326)^2 \cdot (0.35)^2}{(0.1)^2} = 66.28 \approx 67$$

where $Z_{\alpha/2} = \text{invnorm}(0.01, 0, 1) = -2.326$

b. If the amount is not sufficient, hold your confidence level constant and calculate the needed confidence interval width.

$n = 50$

$\sigma = 0.35$ (gm)

$100(1-\alpha)\% = 98\%$

$$n = \frac{(Z_{\alpha/2})^2 * \sigma^2}{B^2} \Rightarrow 50 = \frac{(-2.326)^2 \cdot (0.35)^2}{B^2}$$

$$B^2 = \frac{(-2.326)^2 \cdot (0.35)^2}{50} \Rightarrow B = \pm 0.115$$

CI width $= 2B = 0.23$

c. If the amount is not sufficient, hold your confidence interval width constant and calculate your new confidence level.

$n = 50$

$\sigma = 0.35$ (gm)

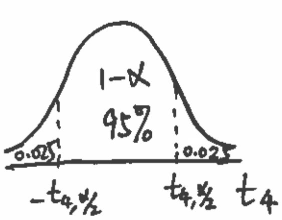
$B = \pm 0.1$ (gm)

$$n = \frac{(Z_{\alpha/2})^2 * \sigma^2}{B^2} \Rightarrow 50 = \frac{Z_{\alpha/2}^2 * (0.35)^2}{(0.1)^2}$$

$$\Rightarrow Z_{\alpha/2}^2 = \frac{50 \cdot (0.1)^2}{(0.35)^2} \Rightarrow Z_{\alpha/2} = \pm 2.02$$

$\frac{\alpha}{2} = \text{normcdf}(-\infty, -2.02, 0, 1) = 0.022$ $\quad CL = 100(1-\alpha)\% = 95.6\%$

2. In a field study one wished to obtain a 95% confidence interval to estimate the mean leaf length on a particular species' mature leaves. They want a maximum error of 2mm. Because they have no idea what the population standard deviation is, a pilot sample of 5 leaves was taken; the sample standard deviation of these 5 leaf lengths was 4.1 mm. How many more leaves do they need to sample?



X = leaf length (mm)

$B = 2$ (mm)

$S = 4.1$ (mm)

$n' = 5$

$100(1-\alpha)\% = 95\%$

$\text{invt}(0.025, 4) = -2.776$

$$n = \frac{(t_{n-1,\alpha/2})^2 \cdot S^2}{B^2} = \frac{(t_{4,0.025})^2 \cdot (4.1)^2}{2^2}$$

$$= \frac{(-2.776)^2 \cdot (4.1)^2}{2^2} = 32.39 \approx 33.$$

Since we have already had 5 leaves,
We need $33 - 5 = 28$ leaves more.

b. If budget restrictions allow only a total sample of size 20 and they still want to be 95% confident, what is the new maximu error?

$n = 20$ (leaves)

$100(1-\alpha)\% = 95\%$

$t_{4,0.025} = -2.776$

$n' = 5$

$S = 4.1$ (mm)

$$n = \frac{(t_{n'-1,\alpha/2})^2 \cdot s^2}{B^2} \Rightarrow 20 = \frac{(-2.776)^2 \cdot (4.1)^2}{B^2}$$

$$\Rightarrow B^2 = \frac{(-2.776)^2 (4.1)^2}{20} \Rightarrow B = \pm 2.55$$

c. If again the total sample size has to be 20 leaves and the maximum error must remain at 2mm, what must the confidence level be?

$n = 20$ (leaves)

$B = 2$ (mm)

$S = 4.1$ (mm)

$n' = 5$

$$n = \frac{(t_{n'-1,\alpha/2})^2 \cdot S^2}{B^2} \Rightarrow 20 = \frac{(t_{4,\alpha/2})^2 \cdot (4.1)^2}{2^2}$$

$$\Rightarrow t_{4,\alpha/2}^2 = \frac{20 \cdot (2^2)}{(4.1)^2} \Rightarrow t_{4,\alpha/2} = \pm 2.182$$

$$tcdf(-2.182, 2.182, 4) = 0.9053$$

So. $100(1-\alpha)\% = 90.53\%$.

Confidence Intervals for the Population Mean

1. A car repair business want to know what is the cost of parts for all of his repair jobs. He randomly selects the receipts for parts purchased on 94 different jobs. The sample mean cost of the parts for these 94 jobs was \$31.32 and the sample standard deviation is \$30.42. Construct a 90% confidence interval for the true mean cost of parts for all of his repair jobs.

X:____

Assumptions:________________________________

____

____

Reliability Coefficient:_________________________

(____)

Interpretation:

2. A quality control engineer in a bakery good plant needs to estimate the mean weight of bags of potato chips which are packed by particular machine. He knows from experience that the population standard deviation is assumed to be 0.1 ounce for this particular machine. He takes a random sample of 36 bags of chips and finds that the sample mean weight of these bags is 16.01 ounces. Construct a 95% confidence interval for the true mean weight of potato chip bags which are packed by this machine.

X:______________________________________

Assumptions:______________________________

____

____

Reliability Coefficient:________________________

(_______________________________________)

Interpretation:

3. The U.S. Bureau of Labor Statistics collects and publishes data on employment and hourly earnings in the aircraft industry. Suppose a random sample of 30 people working in the aircraft industry are selected and their mean hourly earnings from this sample was \$12.01 per hour and the sample standard deviation is \$3.25 per hour. Construct a 99% confidence interval for the true mean hourly rate for persons in the aircraft industry.

X:____

Assumptions:________________________________

____

____

Reliability Coefficient:__________________________

(____)

Interpretation:

4. The *Journal of the American Medical Association* reported on the results of a National Health Interview Survey designed to determine the prevalence of smoking among U.S. adults. Current smokers (11,000 adults in the sample survey) were asked: "on the average, how many cigarettes do you now smoke a day?" The results yielded the following:

Variable	N	Mean	StDev	SE Mean	95.0% C.I.
Smokers	11,000	20.0	15.05	0.15	(19.7, 20.3)

a. Interpret the 95% confidence interval.

b. A tobacco industry researcher claims that the mean number of cigarettes smoked per day by regular cigarette smokers is 15. Comment on this claim.

Confidence Intervals and Hypothesis Testing

1. Everyone who owns or has access to a computer probably has several passwords for various programs. Some psychologists believe passwords are predictable based on personality traits, and many computer hackers claim that any password can be *acquired*. In order to study computer security at Spectaguard Acquisition, a special program was developed and used to systematically try various passwords for randomly selected user accounts. A random sample of time (in hours) needed to obtain each password is given below. Find the 94% confidence interval for the true mean time.

1.88	1.71	2.09	6.60	2.28	3.52	2.64
4.94	1.78	4.13	2.55	1.66	0.28	4.02
4.47	0.68	5.67	0.03	1.89	1.76	2.10
6.58	2.30	3.50	2.66	4.92	1.80	4.11
2.57	1.64	0.26	4.00	4.49	0.66	5.69

X:____

Assumptions:________________________________

____

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____

Reliability Coefficient:__________________________

(____)

If the assumption is, on average, that it takes 3.5 hours to obtain each password, test this assumption using the confidence interval computed above. Explain why you reached your conclusion.

H_o______________________________________

H_a______________________________________

Conclusion______________________________

____

____

Why?___________________________________

____

2. Most major league baseball parks have a device to measure the speed of every pitch. The results are often displayed on a scoreboard and tracked by coaches. A random sample of pitches made during the first inning of baseball games around the country was selected. The speed of each pitch (in mph) is given below. Test at the 5% significance level to see if the mean speed is faster than the previous season mean of 84.25 mph. Assume the data is normally distributed.

85	90	82	86	83	88	87	90	92	84	92	90	89
90	89	84	87	92	89	86	89					

X:____

Assumptions:________________________________

____

____

____

H_o____

H_a____

Test Statistic________________________________

Rejection Region______________________________

Conclusion___________________________________

____

____

P-Value_____________________________________

3. The mean income per year (in dollars) of employees who produce internal and external newsletters and magazines for corporation was reported to be \$51,500 in 2006. These editors and designers work on corporate publications, but not on marketing materials. Due to poor economic conditions and over-supply over the past two years, these corporate communications workers may be experiencing a decrease in salary. A random sample of 38 corporate communications workers revealed a sample mean of \$49, 762 with a sample standard deviation of \$3749. Test at the 5% significance level to determine whether there is any evidence to suggest that the mean income per year of corporate communications workers has decreased.

X:______________________________

Assumptions:__________________________

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____

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H_o______________________________

H_a______________________________

Test Statistic_________________________

Rejection Region_______________________

Conclusion__________________________

____

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P-Value____________________________

Would your conclusion have changed if you had tested at the 1% significance level? Explain.

____

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4. Most patients in need of a kidney transplant are placed on a dialysis machine. There is some evidence to suggest that the longer a patient remains on dialysis, the worse they fare following a transplant. A random sample of kidney transplant patients was obtained, and the wait time (in months) was recorded.

37.3	38.1	23.6	46.2	32.7	19.1	44.1	22.2	30.6	49.1	48.3	33.1	40.0
24.6	19.4	43.7	50.1	33.7	36.5	39.4	34.7	48.0	42.3	23.5		

Assuming normality, construct a 99% confidence interval for the mean wait times.

X:______________________________

Assumptions:______________________________

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Reliability Coefficient:________________________

(____________________________________)

Use the computed confidence interval to see if the data suggests that the mean waiting time for a kidney transplant has changed from the accepted mean time of 45 months. Test at the 1% significance level.

H_o________________________________

H_a________________________________

Conclusion____________________________

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Why?________________________________

5. According to the document *Consumer Expenditures*, a publication of the U.S. Bureau of Labor Statistics, the average consumer unit spent $2749 on apparel and services in 2010. This year, 32 consumer units in the Northeast had the following annual expenditures, in dollars, on apparel and services.

2292	2470	3033	2699	2286	3236	3246	3205	2624	2747	3701	3042
3212	2873	3457	2857	2778	3280	2535	3025	3003	2764	3215	2725
2808	2658	3005	3110	2990	2656	2558	2798				

Test at the 1% significance level to determine if this year's consumer units are spending more on apparel and services compared to that in 2010.

X:____

Assumptions:________________________________

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H_o____

H_a____

Test Statistic________________________________

Rejection Region______________________________

Conclusion__________________________________

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P-Value____________________________________

Based on your conclusion, what type of error have you subjected yourself to and explain what that error entails based on this specific situation.

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Confidence Intervals and Hypothesis Testing

1. Everyone who owns or has access to a computer probably has several passwords for various programs. Some psychologists believe passwords are predictable based on personality traits, and many computer hackers claim that any password can be *acquired*. In order to study computer security at Spectaguard Acquisition, a special program was developed and used to systematically try various passwords for randomly selected user accounts. A random sample of time (in hours) needed to obtain each password is given below. Find the 94% confidence interval for the true mean time.

1.88	1.71	2.09	6.60	2.28	3.52	2.64
4.94	1.78	4.13	2.55	1.66	0.28	4.02
4.47	0.68	5.67	0.03	1.89	1.76	2.10
6.58	2.30	3.50	2.66	4.92	1.80	4.11
2.57	1.64	0.26	4.00	4.49	0.66	5.69

X:______________________________________

Assumptions:______________________________

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Reliability Coefficient:________________________

(______________________________________)

If the assumption is, on average, that it takes 3.5 hours to obtain each password, test this assumption using the confidence interval computed above. Explain why you reached your conclusion.

H_o____________________________________

H_a____________________________________

Conclusion______________________________

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Why?___________________________________

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2. The mean income per year (in dollars) of employees who produce internal and external newsletters and magazines for corporation was reported to be \$51,500 in 2006. These editors and designers work on corporate publications, but not on marketing materials. Due to poor economic conditions and over-supply over the past two years, these corporate communications workers may be experiencing a decrease in salary. A random sample of 38 corporate communications workers revealed a sample mean of \$49, 762 with a sample standard deviation of \$3749. Test at the 5% significance level to determine whether there is any evidence to suggest that the mean income per year of corporate communications workers has decreased.

X:____

Assumptions:________________________________

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H_o____

H_a____

Test Statistic_________________________________

Rejection Region______________________________

Conclusion___________________________________

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P-Value______________________________________

Would your conclusion have changed if you had tested at the 1% significance level? Explain.

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3. According to the document *Consumer Expenditures*, a publication of the U.S. Bureau of Labor Statistics, the average consumer unit spent \$2749 on apparel and services in 2010. This year, 32 consumer units in the Northeast had the following annual expenditures, in dollars, on apparel and services.

2292	2470	3033	2699	2286	3236	3246	3205	2624	2747	3701	3042
3212	2873	3457	2857	2778	3280	2535	3025	3003	2764	3215	2725
2808	2658	3005	3110	2990	2656	2558	2798				

Test at the 1% significance level to determine if this year's consumer units are spending more on apparel and services compared to that in 2010.

X:______________________________

Assumptions:______________________________

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H_o______________________________

H_a______________________________

Test Statistic______________________________

Rejection Region______________________________

Conclusion______________________________

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P-Value______________________________

Based on your conclusion, what type of error have you subjected yourself to and explain what that error entails based on this specific situation.

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Studentized T-Distribution

1. A large university reports that the mean salary of the parents of an entering class is $91,600. To see how this compares to his university, a presidents surveys 28 randomly selected families and finds that their mean income is $78,050 with a standard deviation of $4988. Assuming the incomes are normally distributed, first estimate the true mean parental income by constructing a 95% confidence interval.

X:____

Assumptions:________________________________

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Reliability Coefficient:_________________________

(____)

Interpretation:

Based on the confidence interval, is there sufficient evidence at the 5% significance level for the president to conclude that a difference exists between his university and the large university?

H_o:____

H_a:____

Conclusion:__________________________________

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Why?______________________________________

2. Postmortem interval (PMI) is the elapsed time between death and the performance of an autopsy on the cadaver. Brain and Language (June 1995) reported on the PMIs (hours) of 22 randomly selected human brain specimens obtained at autopsy. The analysis of the data is shown below.

Variable	N	Mean	StDev	SE Mean	95% CI
PMI	22	7.300	3.185	0.679	(5.888, 8.712)

The acceptable PMI is 7.0 hours. Does this data suggest that the PMI differs from this level? Assume normality and test at the 5% significance level.

X:____

Assumptions:________________________________

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H_o:____

H_a:____

Conclusion:_________________________________

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Why?______________________________________

3. A consumer protection group is concerned that a ketchup manufacturer is filling its 20-oz family size containers with less than 20 oz of ketchup. The group randomly purchases 10 family size bottles of this ketchup, weighs the contents of each, and finds the mean weight equal to 19.86 oz and the sample standard deviation equal to 0.22 oz. Do the data provide sufficient evidence for the consumer group to conclude that the mean fill per family-size bottle is less than 20 oz at the 5% significance level? Assume the weights follow a normal distribution.

X:____________________________________

Assumptions:____________________________

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H_o____________________________________

H_a____________________________________

Test Statistic_____________________________

Rejection Region___________________________

Conclusion_______________________________

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P-Value__________________________________

If the test were conducted on a periodic basis by the company's quality control department, is the consumer group more concerned about making a Type I or a Type II error?

The ketchup company is also interested in the mean amount of ketchup per bottle. It does not wish to overfill them. Which type of error is more serious from the company's point of view?

4. Although it is known that the white shark grows to a mean length of 21 feet, a marine biologist believes that the great white shark off the Bermuda coast grows much longer due to unusual feeding habits. These shark lengths are assumed to follow a normal distribution. To test the claim, a random sample of 5 full grown great white sharks are captured off the Bermuda coast, measured, and then set free. The sample mean length is 22.4 feet and the sample standard deviation is 1.58 feet. Do the data provide sufficient evidence to support the marine biologist's claim at the 10% significance level?

X:____

Assumptions:________________________________

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H_o____

H_a____

Test Statistic________________________________

Rejection Region______________________________

Conclusion__________________________________

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P-Value____________________________________

What type of error have you subjected yourself to? Explain in terms of this particular problem.

Would your conclusion have changed if you had tested at the 5% significance level? Explain.

5. Most water treatment facilities monitor the quality of their drinking water on an hourly basis. One variable monitored is pH. One water treatment plant has a target pH of 8.5. A quality control engineer took a random sample of 17 one-hour test results and came up with a sample mean of 8.42 and a sample variance of 0.0256. Does this data provide sufficient evidence that the mean pH level in the water differs from the target level? Assume normality and test at the 1% significance level.

X:____

Assumptions:________________________________

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H_o____

H_a____

Test Statistic________________________________

Rejection Region______________________________

Conclusion__________________________________

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P-Value____________________________________

6. The mean time between failures (in hours) for a Teletronic company radio used in light aircraft is 420 hours. After 15 new radios were modified in an attempt to improve reliability, tests were conducted to measure the time between failures for these 15. Test at the 5% significance level.

Variable	N	Mean	StDev	SE Mean	T	Pvalue
Hours	15	434.73	18.01	4.65	3.17	0.0034

X:____________________________________

Assumptions:________________________________

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H_o____________________________________

H_a____________________________________

Test Statistic_______________________________

Conclusion_________________________________

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P-Value___________________________________

Confidence Intervals and Hypothesis Tests-Unknown Variance and Small Sample Size

1. Most patients in need of a kidney transplant are placed on a dialysis machine. There is some evidence to suggest that the longer a patient remains on dialysis, the worse they fare following a transplant. A random sample of kidney transplant patients was obtained, and the wait time (in months) was recorded.

37.3	38.1	23.6	46.2	32.7	19.1	44.1	22.2	30.6	49.1	48.3	33.1	40.0
24.6	19.4	43.7	50.1	33.7	36.5	39.4	34.7	48.0	42.3	23.5		

Assuming normality, construct a 99% confidence interval for the mean wait times.

X:____

Assumptions:________________________________

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Reliability Coefficient:__________________________

(____)

Use the computed confidence interval to see if the data suggests that the mean waiting time for a kidney transplant has changed from the accepted mean time of 45 months. Test at the 1% significance level.

H_o__________________________________

H_a___________________________________

Conclusion______________________________

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Why?_________________________________

2. Most major league baseball parks have a device to measure the speed of every pitch. The results are often displayed on a scoreboard and tracked by coaches. A random sample of pitches made during the first inning of baseball games around the country was selected. The speed of each pitch (in mph) is given below. Test at the 5% significance level to see if the mean speed is faster than the previous season mean of 84.25 mph. Assume the data is normally distributed.

85	90	82	86	83	88	87	90	92	84	92	90	89
90	89	84	87	92	89	86	89					

X:____

Assumptions:________________________________

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H_o____

H_a____

Test Statistic__________________________________

Rejection Region______________________________

Conclusion___________________________________

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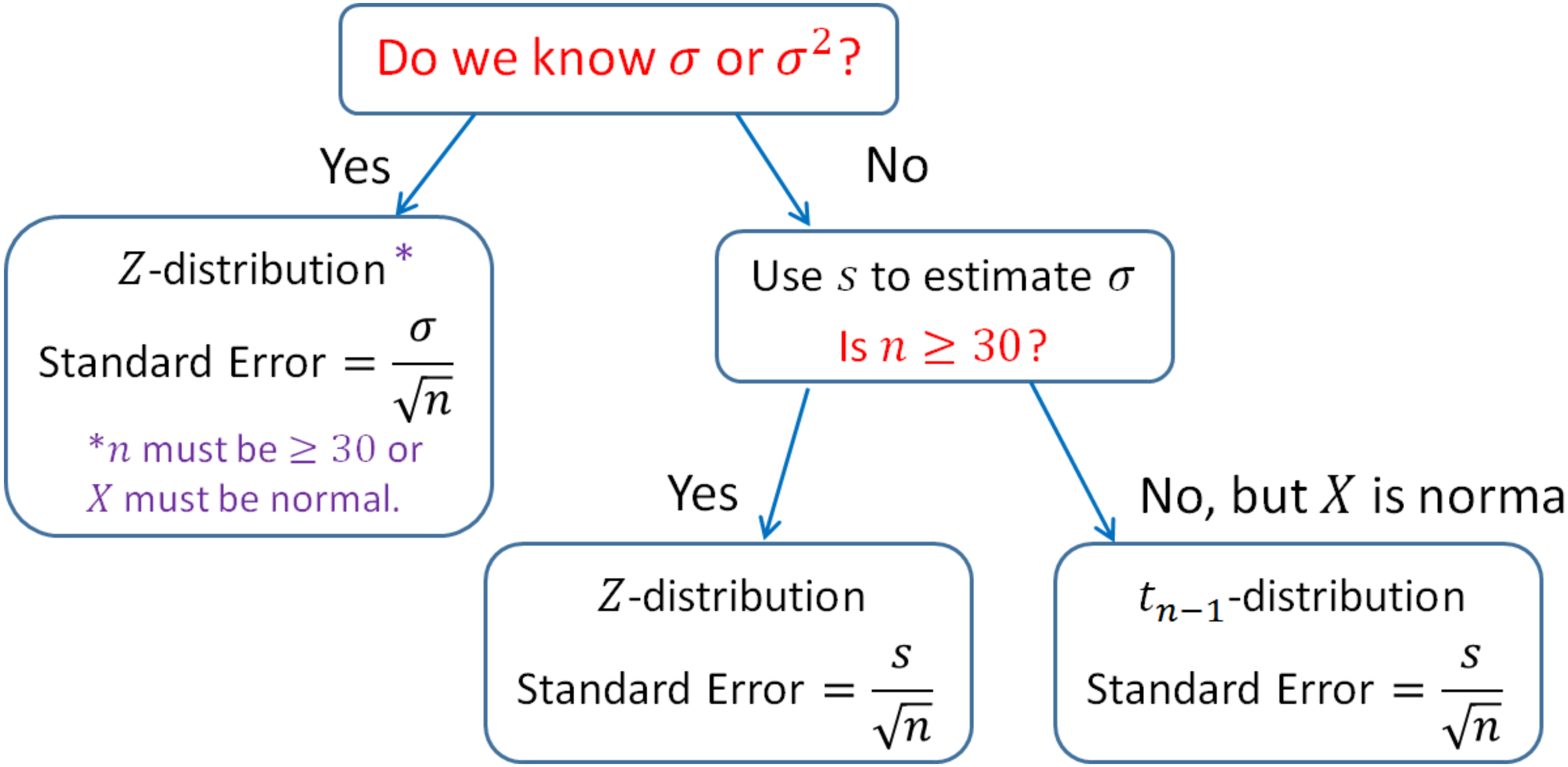
P-Value______________________________________


Do we know σ or σ^2?
Yes
No
Z-distribution*
Standard Error $= \frac{\sigma}{\sqrt{n}}$
*n must be ≥ 30 or
X must be normal.
Use s to estimate σ
Is $n \geq 30$?
Yes
No, but X is normal
Z-distribution
Standard Error $= \frac{s}{\sqrt{n}}$
t_{n-1}-distribution
Standard Error $= \frac{s}{\sqrt{n}}$

Hypothesis Tests For One Population Mean, μ

1. White blood cells are the body's natural defense mechanism against disease and infection. The mean white blood cell count in healthy adults measured as part of a CBC (complete blood count), is approximately 7.5 x 10^3 cells/microliter. A company developing a new drug to treat arthritis pain must check for any side effects. A random sample of 45 patients using the new drug was selected, and the white blood cell count of each patient was measured. The sample mean was 7.77 x 10^3 cells/mcl and the sample standard deviation was 1.21 x 10^3 cells/mcl. Test at the 5% significance level to determine whether there is any change in the mean white blood cell count due to the arthritis drug.

$\mu_0 = 7.5 \times 10^3$ cells/μl

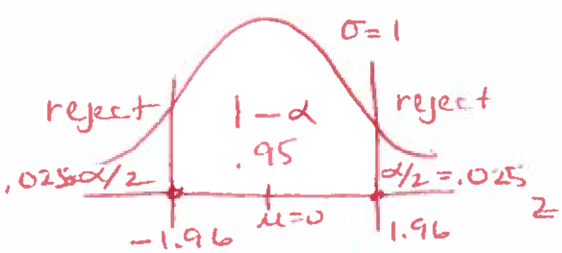
$n = 45$ adults

$\bar{x} = 7.77 \times 10^3$ cells/μl

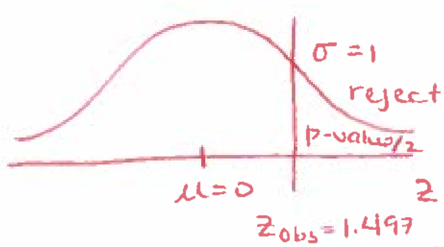
$s = 1.21 \times 10^3$ cells/μl

$\alpha = .05$

$$z_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{(7.77 - 7.5)}{1.21/\sqrt{45}}$$



invnorm(.025,0,1) = −1.96



normcdf(1.497,∞,0,1) = 0.067

Random Variable: white blood cell count (x10^3 cells/μl)

Assumptions: SRS

s is a good estimate of σ

$\bar{x}$ is normally distributed because n is ≥ 30

H_0: $\mu = 7.5 \times 10^3/\mu l$

H_a: $\mu \neq 7.5 \times 10^3/\mu l$

Test Statistic: $z_{obs} = 1.497$

Rejection Region: Reject H_0 if $z_{obs} \geq 1.96$ or $z_{obs} \leq -1.96$

Conclusion: Fail to reject H_0, 95% sure we couldn't prove there was a change in the mean white blood cell count.

P-Value: 0.134

$\frac{p\text{-value}}{2} = .067$ so p-value = 0.134

What type of error could you have subjected yourself to? Explain what this means in terms of this particular situation.

Type II, there was a change in the mean white blood cell count, but we couldn't prove it

---

95% CI for μ

$\bar{x} \pm z_{\alpha/2} * s/\sqrt{n}$

$7.77 \pm 1.96 * 1.21/\sqrt{45}$

7.77 ± 0.35 (7.42, 8.12 x 10^3 cells/μl)

2. A plantation of red pine seems to be growing slower than expected so the forest manager decided to take a random sample of 49 saplings and measure the seasonal diameter growth on these 49 samplings. The mean seasonal diameter growth from the sample was 21 mm and the sample standard deviation for this seasonal diameter growth was 11 mm. Can it be concluded from these data that the mean seasonal diameter growth for this particular plantation is less than the 30 mm of growth that the manager expected? Test at the 5% significance level.

$n = 49$ saplings

$\bar{x} = 21\text{mm}$

$s = 11\text{ mm}$

$\alpha = .05$

$\mu_0 = 30\text{mm}$

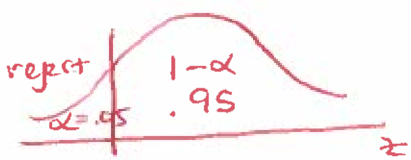
Random Variable: seasonal diameter growth (mm)

Assumptions: SRS

s is a good estimate of σ

$\bar{x}$ is normally distributed because $n \geq 30$

$$z_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{21-30}{11/\sqrt{49}}$$



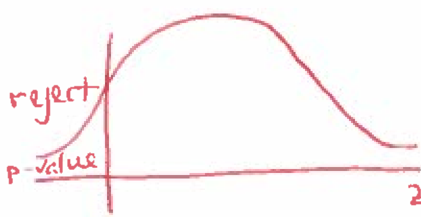
invnorm(.05,0,1) = −1.645

H_0 $\mu = 30\text{mm}$

H_a $\mu < 30\text{mm}$

Test Statistic $z_{obs} = -5.72$

Rejection Region Reject H_0 if $z_{obs} \leq -1.645$



$z_{obs} = -5.72$

normcdf(−∞, −5.72, 0, 1)

Conclusion Reject H_0, 95% sure the mean seasonal diameter growth is less than 30mm for this plantation

P-Value 5.34×10^{-9}

What type of error could you have subjected yourself to? Explain what this means in terms of this particular situation.

Type I - proved the redpine were growing slower than expected when they really weren't

Would your conclusion have changed if you had tested at the 1% significance level? Explain.

No, still reject H_0 because p-value $(5.34 \times 10^{-9}) < \alpha\ (.01)$

3. Today's society is concerned about fat content. Researchers conducted a study on the amount of butterfat in mild to determine if it is greater than 3.5 points which has been deemed an acceptable level. Tests for butterfat content were performed on milk from a random sample of 20 Holstein dairy cows. The sample mean amount of butterfat in the milk was 3.88points. Work in this area has assumed that the population standard deviation is expected to be 0.42 points and the butterfat content for these cows follows a normal distribution. Test at the 10% significance level to determine if this herd of Holstein dairy cows produce milk with a butterfat content which is higher than the acceptable level.

$n = 20$

$\bar{x} = 3.88$

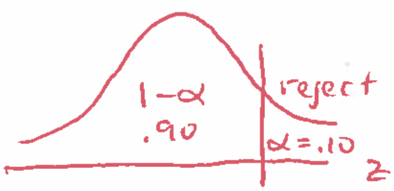
$\sigma = 0.42$

$\alpha = .10$

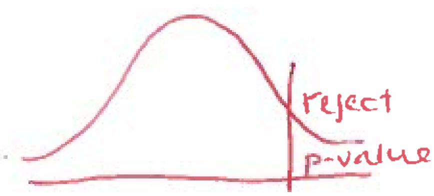
$\mu_0 = 3.5$

$$z_{obs} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{3.88 - 3.5}{0.42/\sqrt{20}}$$

$$= 4.05$$



invnorm(.90, 0, 1) = 1.282



$z_{obs} = 4.05$

normcdf(4.05, ∞, 0, 1)

Random Variable: X = butterfat content in milk (pts)

Assumptions: SRS

σ is known

$\bar{x}$ is normally distributed because butterfat contents are normal

H_0: $\mu = 3.5$ pts

H_a: $\mu > 3.5$ pts

Test Statistic: $z_{obs} = 4.05$

Rejection Region: Reject H_0 if $z_{obs} \geq 1.282$

Conclusion: Reject H_0, 90% sure the mean butterfat content in milk for these cows is more than 3.5 pts

P-Value: 0.000025

4. Research reported in the Journal of Psychology studied the personality characteristics of obese individuals. One variable, the locus of control (LOC) measures the individual's degree of belief that he or she has control over situations. High scores on the LOC scale indicate less perceived control. For one random sample of 49 obese adolescents, the sample data collected is shown summarized in the output below. Suppose we wish to test whether the mean LOC score for all obese adolescents exceeds 10, the average for "normal weight" individuals. Test at the 1% significance.

Variable	N	Mean	StDEV	SE MEAN	Z	PValue
LOC	49	10.89	2.48	0.354	2.512	0.006

Random Variable: X = LOC

Assumptions: SRS

s is a good estimate of σ

$\bar{x}$ is normally distributed because $n \geq 30$

H_o $\mu = 10$

H_a $\mu > 10$

Test Statistic $z_{obs} = 2.512$

p-value(0.006) < α(.01)

Conclusion Reject H_o, 99% sure the mean LOC for these adolescents is higher than 10.

P-Value 0.006

Confidence Intervals and Hypothesis Testing

1. Everyone who owns or has access to a computer probably has several passwords for various programs. Some psychologists believe passwords are predictable based on personality traits, and many computer hackers claim that any password can be *acquired.* In order to study computer security at Spectaguard Acquisition, a special program was developed and used to systematically try various passwords for randomly selected user accounts. A random sample of time (in hours) needed to obtain each password is given below. Find the 94% confidence interval for the true mean time.

1.88	1.71	2.09	6.60	2.28	3.52	2.64
4.94	1.78	4.13	2.55	1.66	0.28	4.02
4.47	0.68	5.67	0.03	1.89	1.76	2.10
6.58	2.30	3.50	2.66	4.92	1.80	4.11
2.57	1.64	0.26	4.00	4.49	0.66	5.69

$n = 35$ $\quad \bar{x} = 2.910$ $\quad s = 1.773$

Random Variable: X = time to get password (hrs)

$\bar{x} \pm z_{\alpha/2} * s/\sqrt{n}$

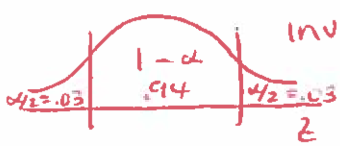
$2.910 \pm 1.881 * 1.773/\sqrt{35}$

2.910 ± 0.564

Assumptions: SRS

s is a good estimate of σ

$\bar{x}$ is normal because $n \geq 30$



invnorm(.03,0,1)

Reliability Coefficient: $z = \pm 1.881$

(2.346 hrs , 3.474 hrs)

If the assumption is, on average, that it takes 3.5 hours to obtain each password, test this assumption using the confidence interval computed above. Explain why you reached your conclusion.

H_o $\mu = 3.5$ hrs

H_a $\mu \neq 3.5$ hrs

Conclusion Reject H_o, 94% sure the mean time to obtain a password at Spectaguard Acquisition is different from 3.5 hrs.

Why? 3.5 falls outside CI

2. Most major league baseball parks have a device to measure the speed of every pitch. The results are often displayed on a scoreboard and tracked by coaches. A random sample of pitches made during the first inning of baseball games around the country was selected. The speed of each pitch (in mph) is given below. Test at the 5% significance level to see if the mean speed is faster than the previous season mean of 84.25 mph. Assume the data is normally distributed.

85	90	82	86	83	88	87	90	92	84	92	90	89
90	89	84	87	92	89	86	89					

$n = 21$ $\quad \bar{x} = 87.8$ $\quad s = 2.99$

Random Variable: X = speed of pitch (mph)

Assumptions: SRS

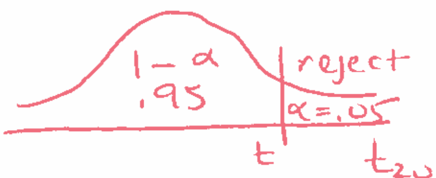
s is a good estimate of σ

$\bar{x}$ is normally distributed because pitch speeds are normal

$$t_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{87.8 - 84.25}{2.99/\sqrt{21}}$$

H_0 $\mu = 84.25$

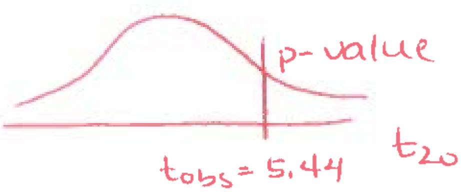
H_a $\mu > 84.25$



invt(.95, 20)

Test Statistic $t_{obs} = 5.44$

Rejection Region Reject H_0 if $t_{obs} \geq 1.725$



tcdf(5.44, ∞, 20)

Conclusion Reject H_0, 95% sure the mean pitch speed for this season is faster than 84.25 mph

P-Value 0.000013

3. The mean income per year (in dollars) of employees who produce internal and external newsletters and magazines for corporation was reported to be \$51,500 in 2006. These editors and designers work on corporate publications, but not on marketing materials. Due to poor economic conditions and over-supply over the past two years, these corporate communications workers may be experiencing a decrease in salary. A random sample of 38 corporate communications workers revealed a sample mean of \$49, 762 with a sample standard deviation of \$3749. Test at the 5% significance level to determine whether there is any evidence to suggest that the mean income per year of corporate communications workers has decreased.

Random Variable: X = income/year (\$)

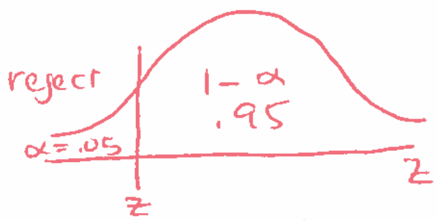
Assumptions: SRS

s is a good estimate of σ

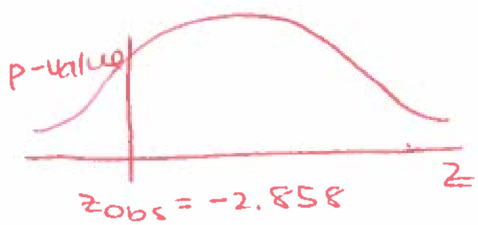
$\bar{x}$ is normally distributed

because $n \geq 30$

$$z_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{49{,}762 - 51{,}500}{3749/\sqrt{38}}$$



invnorm(.05, 0, 1)



normcdf(−∞, −2.858, 0, 1)

H_0: $\mu = \$51,500$

H_a: $\mu < \$51,500$

Test Statistic: $z_{obs} = -2.858$

Rejection Region: Reject H_0 if $z_{obs} \leq -1.645$

Conclusion: Reject H_0, 95% sure the mean income/year of corporate communications workers is less than \$51,500

P-Value: 0.0021

Would your conclusion have changed if you had tested at the 1% significance level? Explain.

No - still reject H_0 because p-value (.0021) is less than α (.01)

4. Most patients in need of a kidney transplant are placed on a dialysis machine. There is some evidence to suggest that the longer a patient remains on dialysis, the worse they fare following a transplant. A random sample of kidney transplant patients was obtained, and the wait time (in months) was recorded.

37.3	38.1	23.6	46.2	32.7	19.1	44.1	22.2	30.6	49.1	48.3	33.1	40.0
24.6	19.4	43.7	50.1	33.7	36.5	39.4	34.7	48.0	42.3	23.5		

Assuming normality, construct a 99% confidence interval for the mean wait times.

$n = 24$ $\bar{x} = 35.85$ $s = 9.81$

$\bar{x} \pm t_{n-1, \alpha/2} * s/\sqrt{n}$

$35.85 \pm 2.807 * 9.81/\sqrt{24}$

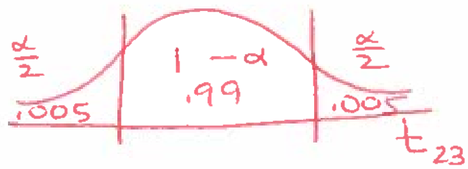
35.85 ± 5.621

Random Variable: X = wait time (months)

Assumptions: SRS

s is a good estimate of σ

$\bar{x}$ is normally distributed because wait times are normal



invt(.005, 23)

Reliability Coefficient: $t = \pm 2.807$

(30.23 months, 41.47 months)

Use the computed confidence interval to see if the data suggests that the mean waiting time for a kidney transplant has changed from the accepted mean time of 45 months. Test at the 1% significance level.

H_o $\mu = 45$ months

H_a $\mu \neq 45$ months

Conclusion Reject H_o, 99% sure the mean waiting time for kidney transplant patients is different from 45 months.

Why? 45 falls outside CI

5. According to the document *Consumer Expenditures*, a publication of the U.S. Bureau of Labor Statistics, the average consumer unit spent \$2749 on apparel and services in 2010. This year, 32 consumer units in the Northeast had the following annual expenditures, in dollars, on apparel and services.

2292	2470	3033	2699	2286	3236	3246	3205	2624	2747	3701	3042
3212	2873	3457	2857	2778	3280	2535	3025	3003	2764	3215	2725
2808	2658	3005	3110	2990	2656	2558	2798				

$n = 32 \quad \bar{x} = 2902.75 \quad s = 324.82$

Test at the 1% significance level to determine if this year's consumer units are spending more on apparel and services compared to that in 2010.

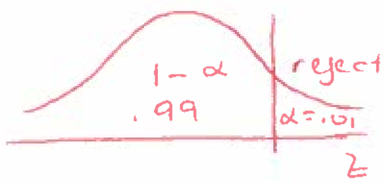
Random Variable: annual expenditures (\$)

$$z_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2902.75 - 2749}{324.82/\sqrt{32}}$$

Assumptions: SRS

s is a good estimate of σ

$\bar{x}$ is normally distributed because $n \geq 30$



H_0: $\mu = \$2749$

H_a: $\mu > \$2749$

invnorm(.99,0,1)

Test Statistic: $z_{obs} = 2.678$

Rejection Region: Reject H_0 if $z_{obs} \geq 2.33$

normcdf(2.678, ∞, 0, 1)

Conclusion: Reject H_0, 99% sure the mean annual expenditures for apparel and services with this year's consumer units is more than \$2749

P-Value: 0.0037

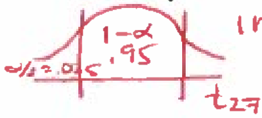
Based on your conclusion, what type of error have you subjected yourself to and explain what that error entails based on this specific situation.

Type I – proved this year's consumer units spend more, on average, on apparel and services than in 2010, but in fact, they really didn't.

Studentized T-Distribution

1. A large university reports that the mean salary of the parents of an entering class is $91,600. To see how this compares to his university, a presidents surveys 28 randomly selected families and finds that their mean income is $78,050 with a standard deviation of $4988. Assuming the incomes are normally distributed, first estimate the true mean parental income by constructing a 95% confidence interval.

$n = 28$ families
$\bar{x} = \$78{,}050$
$s = \$4988$
$100(1-\alpha)\% = 95\%$



$\text{invT}(.025, 27) = -2.052$

Random Variable: parental income ($)

Assumptions: SRS

s is a good estimate of σ
n is small (< 30)
$\bar{x}$ is normally distributed because incomes are normal

Reliability Coefficient: $t_{27} = -2.052$

$\bar{x} \pm t_{n-1,\alpha/2} * s/\sqrt{n}$

$78050 \pm t_{27,.025} * 4988/\sqrt{28}$

$78050 \pm 2.052 * 4988/\sqrt{28}$

78050 ± 1934.3

($76,115.7 , $79,984.3)

Based on the confidence interval, is there sufficient evidence at the 5% significance level for the president to conclude that a difference exists between his university and the large university?

H_0: $\mu = \$91{,}600$

H_a: $\mu \neq \$91{,}600$

Conclusion: Reject H_0, 95% sure the mean parental income for this class is different from $91,600

Why? $91,600 falls outside CI

2. Postmortem interval (PMI) is the elapsed time between death and the performance of an autopsy on the cadaver. Brain and Language (June 1995) reported on the PMIs (hours) of 22 randomly selected human brain specimens obtained at autopsy. The analysis of the data is shown below.

Variable	N	Mean	StDev	SE Mean	95% CI
PMI	22	7.300	3.185	0.679	(5.888, 8.712)

The acceptable PMI is 7.0 hours. Does this data suggest that the PMI differs from this level? Assume normality and test at the 5% significance level.

H_0: $\mu = 7.0$ hrs

H_a: $\mu \neq 7.0$ hrs

Conclusion: Fail to reject H_0, 95% sure we couldn't prove the mean PMI for this coroner is different from 7.0 hrs.

Why? 7.0 falls inside CI

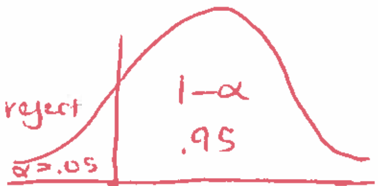
3. A consumer protection group is concerned that a ketchup manufacturer is filling its 20-oz family size containers with less than 20 oz of ketchup. The group randomly purchases 10 family size bottles of this ketchup, weighs the contents of each, and finds the mean weight equal to 19.86 oz and the sample standard deviation equal to 0.22 oz. Do the data provide sufficient evidence for the consumer group to conclude that the mean fill per family-size bottle is less than 20 oz at the 5% significance level? Assume the weights follow a normal distribution.

$n = 10$ bottles
$\bar{x} = 19.86$ oz
$s = 0.22$ oz
$\alpha = .05$
$\mu_0 = 20$ oz

Random Variable: amount of ketchup/bottle (oz)

Assumptions: SRS
s is a good estimate of σ
n is small so $\bar{x}$ is normally distributed because the weights are normal

$$t_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{19.86 - 20}{0.22/\sqrt{10}}$$



$t = -1.833$ $\quad t_9$

invnorm(.05, 9) = −1.833

H_0 $\mu = 20$ oz

H_a $\mu < 20$ oz

Test Statistic $t_{obs} = -2.01$

Rejection Region Reject H_0 if $t_{obs} \leq -1.833$

$t_{obs} = -2.01$ $\quad t_9$

tcdf(−∞, −2.01, 9) = 0.038

Conclusion Reject H_0, 95% sure the mean amount of ketchup/bottle coming from this manufacturer is less than 20 oz.

P-Value 0.038

If the test were conducted on a periodic basis by the company's quality control department, is the consumer group more concerned about making a Type I or a Type II error?

Type II

The ketchup company is also interested in the mean amount of ketchup per bottle. It does not wish to overfill them. Which type of error is more serious from the company's point of view?

Type I

Type I = P(reject H_0 | H_0 is true) – proved the manufacturer is underfilling when he really wasn't

Type II = P(fail to reject H_0 | H_0 is false) – the manufacturer is underfilling but couldn't prove it.

4. Although it is known that the white shark grows to a mean length of 21 feet, a marine biologist believes that the great white shark off the Bermuda coast grows much longer due to unusual feeding habits. These shark lengths are assumed to follow a normal distribution. To test the claim, a random sample of 5 full grown great white sharks are captured off the Bermuda coast, measured, and then set free. The sample mean length is 22.4 feet and the sample standard deviation is 1.58 feet. Do the data provide sufficient evidence to support the marine biologist's claim at the 10% significance level?

$n = 5$ sharks

$\bar{x} = 22.4$ ft

$s = 1.58$ ft

$\alpha = 0.10$

$\mu_0 = 21$ ft

Random Variable: X = length of shark (ft)

Assumptions: SRS

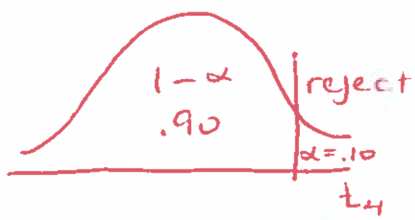
s is a good estimate of σ

$\bar{x}$ is normally distributed because lengths are normal

$$t_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{22.4 - 21}{1.58/\sqrt{5}}$$

H₀: $\mu = 21$ ft

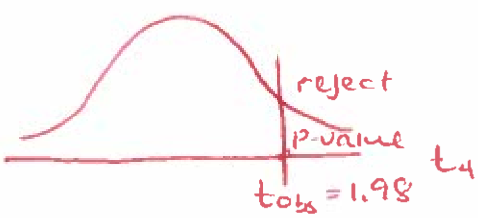
Hₐ: $\mu > 21$ ft



invt(.90, 4) = 1.533

Test Statistic: $t_{obs} = 1.98$

Rejection Region: Reject H₀ if $t_{obs} \geq 1.533$



tcdf(1.98, ∞, 4)

Conclusion: Reject H₀, 90% sure the mean length of shark's off the Bermuda coast is more than 21ft (they are longer)

P-Value: 0.059

What type of error have you subjected yourself to? Explain in terms of this particular problem.

Type I - proved they were growing longer when they really weren't

Would your conclusion have changed if you had tested at the 5% significance level? Explain.

Yes, would fail to reject

p-value (.059) > α (.05)

6. The mean time between failures (in hours) for a Teletronic company radio used in light aircraft is 420 hours. After 15 new radios were modified in an attempt to improve reliability, tests were conducted to measure the time between failures for these 15. Test at the 5% significance level.

Variable	N	Mean	StDev	SE Mean	T	Pvalue
Hours	15	434.73	18.01	4.65	3.17	0.0034

No mention of normality but for this to be valid..

Random Variable: X = time between failures (h

Assumptions: SRS

s is a good estimate of σ

$\bar{x}$ is normally distributed because failure times are normal

H_o $\mu = 420$ hr

H_a $\mu > 420$ hrs

Test Statistic $t_{obs} = 3.17$

p-value (.0034) < α (.05)

Conclusion Reject H_o, 95% sure the mean failure time for modified radios is more than 420 hrs.

P-Value 0.0034

+

Hypothesis Testing

The Computer-Assisted Hypnosis Scale (CAHS) is designed to measure a person's susceptibility to hypnosis. CAHS scores range from 0 (no susceptibility to hypnosis) to 12 (extremely high susceptibility to hypnosis). Psychological Assessment reported that University of Tennessee undergraduates had a mean CAHS score of 4.6. Suppose you want to test whether undergraduates at your university are more susceptible to hypnosis than University of Tennessee undergraduates.

a. Set up H_o and H_a for this test.

b. Describe a Type I error for this test.

c. Describe a Type II error for this test.

Traffic on Route 95 in New Hampshire and Maine is slowed by accidents, bad weather, and of course, toll booths. Department of Transportation (DOT) officials have decided to conduct a hypothesis test and will add additional toll booths if there is evidence to suggest that the mean number of cars passing through the Dover Toll Booth Plaza is more than 40,000 vehicles per day.

a. Write the null and alternative hypotheses about μ, the mean number of cars passing through the Dover Plaza.

b. If a type I error is committed, who is more angry, the DOT officials or drivers, and why?

c. If a type II error is committed, who is more angry, the DOT officials or drivers, and why?

Hypothesis Tests For One Population Mean, μ

1. White blood cells are the body's natural defense mechanism against disease and infection. The mean white blood cell count in healthy adults measured as part of a CBC (complete blood count), is approximately 7.5×10^3 cells/microliter. A company developing a new drug to treat arthritis pain must check for any side effects. A random sample of 45 patients using the new drug was selected, and the white blood cell count of each patient was measured. The sample mean was 7.77×10^3 cells/mcl and the sample standard deviation was 1.21×10^3 cells/mcl. Test at the 5% significance level to determine whether there is any change in the mean white blood cell count due to the arthritis drug.

X:____

Assumptions:________________________________

____

____

____

H_o____

H_a____

Test Statistic_________________________________

Rejection Region______________________________

Conclusion___________________________________

____

____

P-Value_____________________________________

What type of error could you have subjected yourself to? Explain what this means in terms of this particular situation.

2. A plantation of red pine seems to be growing slower than expected so the forest manager decided to take a random sample of 49 saplings and measure the seasonal diameter growth on these 49 samplings. The mean seasonal diameter growth from the sample was 21 mm and the sample standard deviation for this seasonal diameter growth was 11 mm. Can it be concluded from these data that the mean seasonal diameter growth for this particular plantation is less than the 30 mm of growth that the manager expected? Test at the 5% significance level.

X:____

Assumptions:________________________________

____

____

____

H_o____

H_a____

Test Statistic________________________________

Rejection Region______________________________

Conclusion__________________________________

____

____

P-Value_____________________________________

What type of error could you have subjected yourself to? Explain what this means in terms of this particular situation.

Would your conclusion have changed if you had tested at the 1% significance level? Explain.

3. Today's society is concerned about fat content. Researchers conducted a study on the amount of butterfat in milk to determine if it is greater than 3.5 points which has been deemed an acceptable level. Tests for butterfat content were performed on milk from a random sample of 20 Holstein dairy cows. The sample mean amount of butterfat in the milk was 3.88points. Work in this area has assumed that the population standard deviation is expected to be 0.42 points and the butterfat content for these cows follows a normal distribution. Test at the 10% significance level to determine if this herd of Holstein dairy cows produce milk with a butterfat content which is higher than the acceptable level.

X:____

Assumptions:________________________________

____

____

____

H_o____

H_a____

Test Statistic________________________________

Rejection Region______________________________

Conclusion__________________________________

____

____

P-Value____________________________________

4. Research reported in the Journal of Psychology studied the personality characteristics of obese individuals. One variable, the locus of control (LOC) measures the individual's degree of belief that he or she has control over situations. High scores on the LOC scale indicate less perceived control. For one random sample of 49 obese adolescents, the sample data collected is shown summarized in the output below. Suppose we wish to test whether the mean LOC score for all obese adolescents exceeds 10, the average for "normal weight" individuals. Test at the 1% significance.

Variable	N	Mean	StDEV	SE MEAN	Z	PValue
LOC	49	10.89	2.48	0.354	2.512	0.006

X:____

Assumptions:________________________________

____

____

____

H_o____

H_a____

Test Statistic__________________________________

Conclusion____________________________________

____

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P-Value______________________________________

Optimal Sample Size

1. Suppose you wish to estimate the true mean weight of the liver of full-grown frogs of a certain variety correct to within plus or minus 0.1 g. Each frog costs $6 and there is $300 available for purchase. If frog liver weights are normally distributed and a population standard deviation of 0.35 g is assumed and you want to be 98% confident, is the amount allotted sufficient?

b. If the amount is not sufficient, hold your confidence level constant and calculate the needed confidence interval width.

c. If the amounts is not sufficient, hold your confidence interval width constant and calculate your new confidence level.

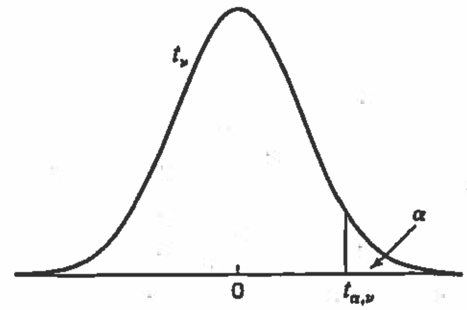
2. In a field study one wished to obtain a 95% confidence interval to estimate the mean leaf length on a particular species' mature leaves. They want a maximum error of 2 mm. Because they have no idea what the population standard deviation is, a pilot sample of 5 leaves was taken; the sample standard deviation of these 5 leaf lengths was 4.1 mm. Assuming the leaf lengths follow a normal distribution, how many more leaves do they need to sample?

b. If budget restrictions allow only a total sample of size 20 and they still want to be 95% confident, what is the new maximum error?

c. If again the total sample size has to be 20 leaves and the maximum error must remain at 2 mm, what must the confidence level be?

Table V Critical Values for the t Distribution

This table contains critical values associated with the t distribution, $t_{\alpha,\nu}$, defined by α and the degrees of freedom, ν.



	α								
ν	0.20	0.10	0.05	0.025	0.01	0.005	0.001	0.0005	0.0001
1	1.3764	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088	636.6192	3183.0988
2	1.0607	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271	31.5991	70.7001
3	0.9785	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145	12.9240	22.2037
4	0.9410	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732	8.6103	13.0337
5	0.9195	1.4759	2.0150	2.5706	3.3649	4.0321	5.8934	6.8688	9.6776
6	0.9057	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076	5.9588	8.0248
7	0.8960	1.4149	1.8946	2.3646	2.9980	3.4995	4.7853	5.4079	7.0634
8	0.8889	1.3968	1.8595	2.3060	2.8965	3.3554	4.5008	5.0413	6.4420
9	0.8834	1.3830	1.8331	2.2622	2.8214	3.2498	4.2968	4.7809	6.0101
10	0.8791	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437	4.5869	5.6938
11	0.8755	1.3634	1.7959	2.2010	2.7181	3.1058	4.0247	4.4370	5.4528
12	0.8726	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296	4.3178	5.2633
13	0.8702	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520	4.2208	5.1106
14	0.8681	1.3450	1.7613	2.1448	2.6245	2.9768	3.7874	4.1405	4.9850
15	0.8662	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328	4.0728	4.8800
16	0.8647	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862	4.0150	4.7909
17	0.8633	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458	3.9651	4.7144
18	0.8620	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105	3.9216	4.6480
19	0.8610	1.3277	1.7291	2.0930	2.5395	2.8609	3.5794	3.8834	4.5899
20	0.8600	1.3253	1.7247	2.0860	2.5280	2.8453	3.5518	3.8495	4.5385
21	0.8591	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272	3.8193	4.4929
22	0.8583	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050	3.7921	4.4520
23	0.8575	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850	3.7676	4.4152
24	0.8569	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668	3.7454	4.3819
25	0.8562	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502	3.7251	4.3517
26	0.8557	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350	3.7066	4.3240
27	0.8551	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210	3.6896	4.2987
28	0.8546	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082	3.6739	4.2754
29	0.8542	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962	3.6594	4.2539
30	0.8538	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852	3.6460	4.2340
40	0.8507	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069	3.5510	4.0942
50	0.8489	1.2987	1.6759	2.0086	2.4033	2.6778	3.2614	3.4960	4.0140
60	0.8477	1.2958	1.6706	2.0003	2.3901	2.6603	3.2317	3.4602	3.9621
70	0.8468	1.2938	1.6669	1.9944	2.3808	2.6479	3.2108	3.4350	3.9257
80	0.8461	1.2922	1.6641	1.9901	2.3739	2.6387	3.1953	3.4163	3.8988
90	0.8456	1.2910	1.6620	1.9867	2.3685	2.6316	3.1833	3.4019	3.8780
100	0.8452	1.2901	1.6602	1.9840	2.3642	2.6259	3.1737	3.3905	3.8616
200	0.8434	1.2858	1.6525	1.9719	2.3451	2.6006	3.1315	3.3398	3.7891
500	0.8423	1.2832	1.6479	1.9647	2.3338	2.5857	3.1066	3.3101	3.7468
∞	0.8416	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190

Population of experiment units

↓

Sampling experiment

↓

Sample of experimental units

↓	↓		↓
Apply factor-level combination 1	Apply factor-level combination 2		Apply factor-level combination *p*
↓	↓		↓
Treatment 1 sample	Treatment 2 sample	…	Treatment *p* sample
↓	↓		↓
Responses for treatment 1	Responses for treatment 2	…	Responses for treatment *p*

A quality control supervisor measures the quality of steel ingots on a scale from 1 to 10. He designs an experiment in which three different temperatures (ranging from 1,100 to 1,200 ºF) and five different pressures (ranging from 500 to 600 psi) are utilized, with 20 ingots produced at each Temperature-Pressure combination. Identify the following elements of the experiment.

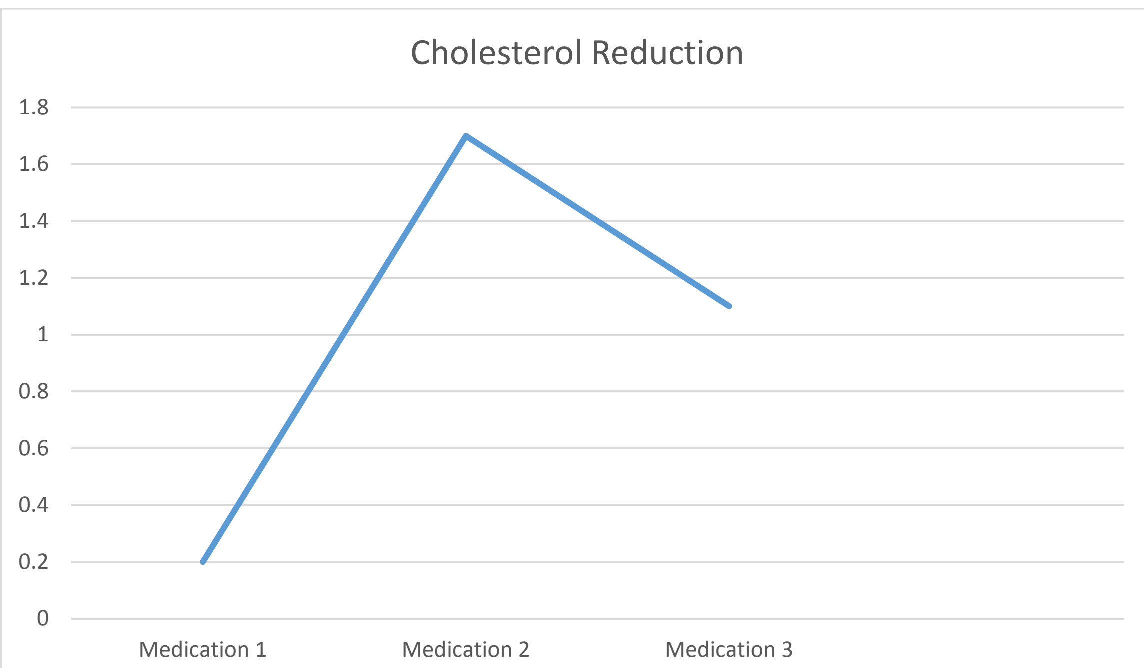
a. Response variable

b. Factor(s) and factor level(s)

c. Treatments

d. Experimental units

A medical researcher wants to compare the effectiveness of three different medications to lower the cholesterol of patients with high blood cholesterol levels. He assigns 60 individuals at random equally to each of the 3 medication treatments and records the reductions in cholesterol for each patient. Below is a figure which displays the sample means. Identify the following elements of the experiment.



a. Response variable

b. Factor and factor level(s)

c. Treatments

d. Experimental units

The Completely Randomized Design

A partially completed ANOVA table for a completely randomized design is shown below:

Source	df	SS	MS	F
Treatments	6	18.4	___	___
Error	__	___	___	
Total	41	45.2		

a. Complete the ANOVA table.

b. How many treatments are involved in the experiment?

c. What is the sample size?

d. Do the data provide sufficient evidence to indicate a difference among the population means? Test at the 5% significance level.

H_o:____

H_a:____

____

Test Statistic:___________________________________

P-Value:______________________________________

Conclusion:____________________________________

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Studies conducted at the University of Melbourne in Australia indicate that there may be a difference between the pain thresholds of blondes and brunettes. Men and women of various ages were divided into four categories according to hair color: light blond, dark blond, light brunette, and dark brunette. The purpose of the experiment was to determine whether hair color is related to the amount of pain evoked by common types of mishaps and assorted types of trauma. Each person in the experiment was given a pain threshold score based on his or her performance in a pain sensitivity test (the higher the score, the higher the person's pain tolerance). SAS was used to conduct the analysis of variance of the data listed below.

Light Blond	Dark Blond	Light Brunette	Dark Brunette
62	63	42	32
60	57	50	39
71	52	41	51
55	41	37	30
48	43	40	35

Sample Mean:

Identify the following:

a. Response Variable

b. Factor and Factor Level(s)

c. Number of Treatments

d. Experimental Units

List the assumptions for a valid analysis of the completely randomized design.

____

____

____

Using the output, test at the 5% significance level to determine whether the mean pain thresholds differ among people possessing the four types of hair color.

Source	df	SS	MS	F	Pvalue
Model	3	1414.50	471.38	7.49	0.0024
Error	16	1006.80	62.93		
Total	19	2421.35			

H_o:______________________________

H_a:______________________________

P-Value:______________________________

Conclusion:______________________________

The next step is to conduct a multiple comparison test.

a. How many confidence intervals do you need?

b. Arrange your sample means in order of magnitude.

c. Conduct a Bonferroni multiple comparison test where the experiment wise error rate is 5%.

d. Display the results of your multiple comparison test.

e. Write your final conclusion.

Conclusion:____

____

____

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Randomized Designs

A new all-purpose cleaner is being test-marketed by placing sales displays in three different locations within various supermarkets. The number of bottles sold from each location within each of the supermarkets tested is reported below.

	Locations		
	I	II	III
	40	32	45
	35	38	48
	44	30	50
	38	35	52
Sample Mean	39.25	33.75	48.75

The following analysis of variance table was constructed for this data.

Source	df	SS	MS	F	P-value
Model	2	460.7	230.4	19.52	.0005
Error	9	106.2	11.8		
Total	11	566.9			

Identify the following:

a. Response Variable

b. Factor and Factor Level(s)

c. Number of Treatments

d. Experimental Units

List the assumptions for a valid analysis of a completely randomized design

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Using the output, test at the 2.5% significance level to determine if there is any difference in the mean number of bottles sold based on location.

H_o:____

H_a:____

____

____

P-Value:_____________________________________

Conclusion:__________________________________

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The next step is to conduct a multiple comparison test.

a. How many confidence intervals do you need?

b. Arrange your sample means in order of magnitude.

c. Conduct a Bonferroni multiple comparison test where the experiment wise error rate is 0.025.

d. Display the results of your multiple comparison test.

e. Write your final conclusion.

Conclusion:____

The USGA wants to compare the mean distances (yards) associated with four different brands of golf balls (A, B, C, and D) when struck by a driver. The output on the following page is the analysis of these data. Test for a difference among brands of golf balls at the 5% significance level.

Response Variable:____________________

Factor:__________________________

H_o:____________________________

H_a:____________________________

Conclusion:______________________

Why?____________________________

The study used Tukey's multiple comparison test with a 5% experiment wise error rate.

a. How many confidence intervals are needed?

b. The means are arranged in order of magnitude from the shortest distance to the longest distance.

Brand:	D	A	B	C

c. Display the results of the analysis and write a full conclusion.

Conclusion:______________________

Analysis of Variance Procedure

Dependent Variable: DISTANCE

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	2794.388750	931.462917	43.99	0.0001
Error	36	762.301000	21.175028		
Corrected Total	39	3556.689750			

R-Square	C.V.	Root MSE	DISTANCE Mean
0.785671	1.785118	4.601633	257.777500

Source	DF	Anova SS	Mean Square	F Value	Pr > F
BRAND	3	2794.388750	931.462917	43.99	0.0001

Tukey's Studentized Range (HSD) Test for variable: DISTANCE

NOTE: This test controls the type I experimentwise error rate.

Alpha= 0.05 Confidence= 0.95 df= 36 MSE= 21.17503
Critical Value of Studentized Range= 3.809
Minimum Significant Difference= 5.5424

Comparisons significant at the 0.05 level are indicated by '***'.

BRAND Comparison	Simultaneous Lower Confidence Limit	Difference Between Means	Simultaneous Upper Confidence Limit	
C - B	3.348	8.890	14.432	***
C - A	13.628	19.170	24.712	***
C - D	15.088	20.630	26.172	***
B - C	-14.432	-8.890	-3.348	***
B - A	4.738	10.280	15.822	***
B - D	6.198	11.740	17.282	***
A - C	-24.712	-19.170	-13.628	***
A - B	-15.822	-10.280	-4.738	***
A - D	-4.082	1.460	7.002	
D - C	-26.172	-20.630	-15.088	***
D - B	-17.282	-11.740	-6.198	***
D - A	-7.002	-1.460	4.082	

Suppose that an appliance manufacturer is interested in determining whether the brand of laundry detergent used affects the amount of dirt (mg) removed from standard household laundry loads. In particular, the manufacturer wants to compare four different brands of detergent (Whisk, Cheer, Tide, Gain). The analysis of variance for this study is below. Test at the 5% significance level.

Source	df	SS	MS	F	Pvalue
Model	3	123.18	41.06	6.92	0.001
Error	36	213.50	5.93		
Total	39	336.68			

Response Variable:____________________

Factor:____________________

H_o:____________________

H_a:____________________

____

____

Conclusion:____________________

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Why?____________________

A multiple comparison test with a 5% experiment error rate yielded the following:

Mean Amount of Dirt (mg) Removed	13.9	14.9	17.2	18.3
Brand of Detergent	Gain	Cheer	Tide	Whisk

Conclusion:____________________

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Completely Randomized Design (One Factor)

Do hypothesis test for difference in k means

- Fail to Reject
  - Stop
- Reject
  - Do multiple comparison test among k means

Complete Factorial Design (Two Factors)

Do hypothesis test for interaction

- Fail to Reject
  - Do main effects hypothesis test for each Factor
    - Factor A
      - Fail to Reject
        - Stop
      - Reject
        - Do multiple comparison test for means in all levels of A
    - Factor B
      - Fail to Reject
        - Stop
      - Reject
        - Do multiple comparison test for means in all levels of B
- Reject
  - Do multiple comparison test among k means

Tests:

H_o: no interaction between factors A and B
H_a: factors A and B interact

H_o: no difference among means for main effect (factor) A
H_a: at least 2 of the means for main effect (factor) A differ

H_o: no difference among means for main effect (factor) B
H_a: at least 2 of the means for main effect (factor) B differ

ANOVA Table:

Source	df	SS	MS	F
Treatment	ab-1	SST	MST	MST/MSE
Error	n-ab*	SSE	MSE	
Corrected Total	n-1*	SS_{TOTAL}		

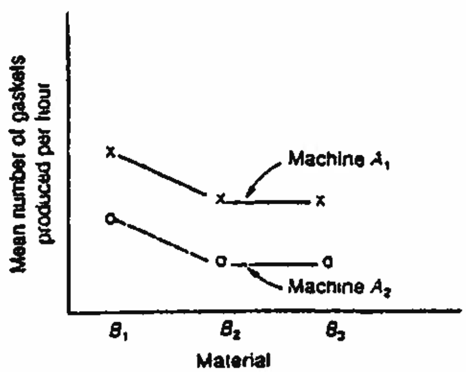
Source	df	SS	MS	F
A	a-1	SS(A)	MS(A)	MS(A)/MSE
B	b-1	SS(B)	MS(B)	MS(B)/MSE
Interaction (AxB)	(a-1)(b-1)	SS(AB)	MS(AB)	MS(AB)/MSE

*n is total sample size

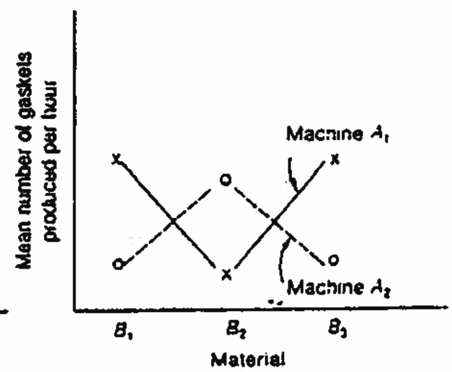
Factorial Experiments

A company that stamps gaskets out of sheets of rubber, plastic, and cork wants to compare the mean number of gaskets produced per hour for two different types of stamping machines. Practically, the manufacturer wants to determine whether one machine is more productive than the other and, even more important, whether one machine is more productive in producing rubber gaskets while the other is more productive in producing plastic or cork gaskets. To answer these questions, the manufacturer decides to conduct an experiment using three types of gasket material, B_1, B_2, and B_3, with each of the two types of stamping machines, A_1 and A_2. Each machine is operated for three 1-hour time periods assigned to six machine-material combinations in random order. The purpose of the randomization is to eliminate the possibility that uncontrolled environmental factors might bias the results. This is a 2x3 factorial design.

Below are hypothetical plots of the means for the six machine-material combinations.



a. No interaction



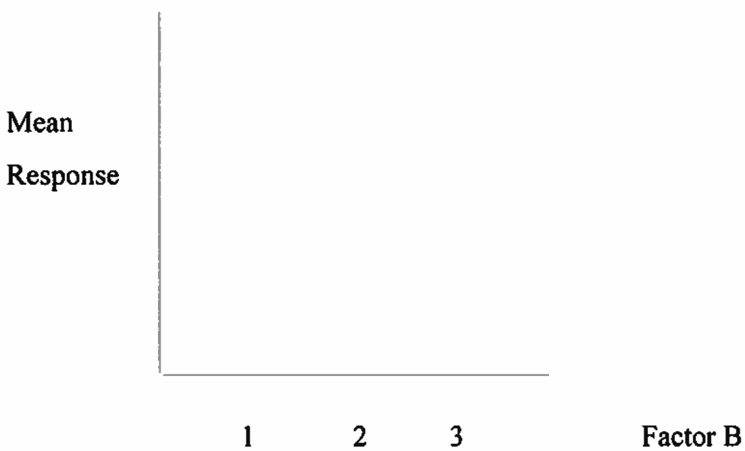
b. Interaction

The following two-way table gives data with two observations for each factor-level combination.

Factor A	Factor B 1	2	3
1	3.1, 4.0	4.6, 4.2	6.7, 7.1
2	5.9, 5.3	2.9, 2.2	3.3, 2.5

a. How many treatments are there and list them.

b. Plot the treatment means below. Use the levels of factor A as plotting symbols. Do the factors appear to interact?



c. Fill in the ANOVA table below.

Source	df	SS	MS	F
A	1	4.441	4.441	___
B	2	4.127	2.063	___
AxB	2	18.007	9.003	___
Error	6	1.475	0.246	
Total	11	28.050		

d. Test to determine whether there is a significant interaction between the two factors at the 5% significance level.

H_o: ______________________________

H_a: ______________________________

Pvalue: ______________________________

Conclusion: ______________________________

____

____

e. Based on your results above, what would be your next step?

Complete factorial designs are commonly employed in marketing research to evaluate the effectiveness of sales strategies. At one supermarket, two of the factors were Price level (regular, reduced price, cost to supermarket) and Display level (normal display space, normal display space plus end-of aisle display, twice the normal display space). Each of the treatments was applied three times to a particular product for a full week and the unit sales ($) for the week was recorded.

a. What is the dependent variable? ____________________

b. What type of factorial experiment is this? ____________________

c. How many treatments are there? ____________________

d. What are the assumptions for this analysis?

____

____

____

The resulting analysis is shown below:

Source	df	SS	MS	F	Pr > F
Model	8	5291151.19	661393.90	1336.85	0.0001
Error	18	8905.33	494.74		
Total	26	5300056.52			

R-Square	C.V.	Root MSE	SALES Mean
0.998320	1.4344689	22.2428	1550.59259

Source	df	Anova SS	MS	F	Pr > F
DISPLAY	2	1691392.5	845696.3	1709.37	0.0001
PRICE	2	3089053.9	1544526.9	3121.89	0.0001
DISPLAYxPRICE	4	510704.8	127676.2	258.07	0.0001

e. Test to determine whether there is a significant interaction between the two factors at the 5% significance level.

H_o:____

H_a:____

Pvalue:______________________________________

Conclusion:___________________________________

____

____

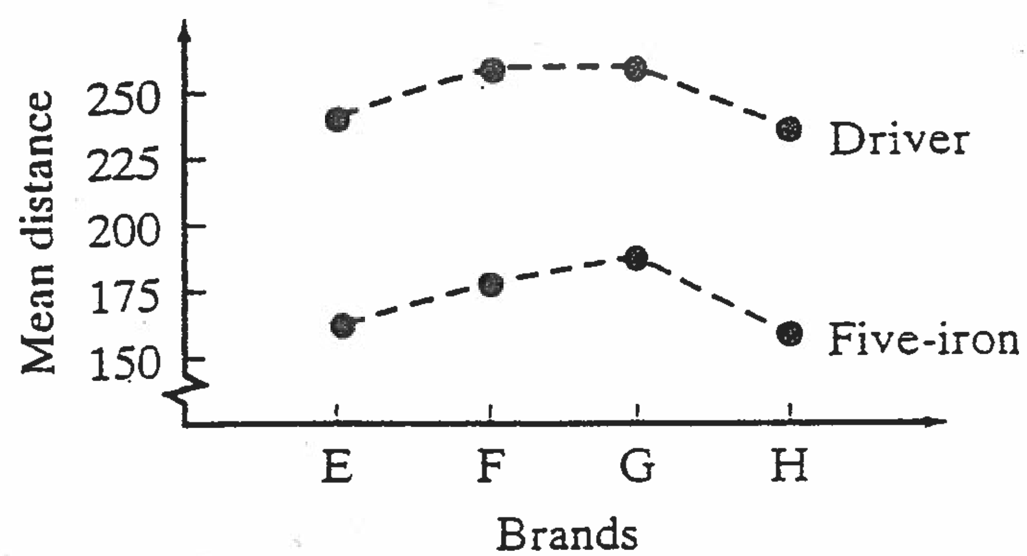
f. Are the tests of the main effects for Price and Display warranted? Explain.

g. Which pairs of treatment means should be compared?

Suppose the USGA tests four different brands of golf balls (E, F, G, H) and two different clubs (driver, five-iron) in a 4x2 factorial design. Each of the eight Brand-Club combinations (treatments) is randomly and independently assigned to four experimental units, each consisting of a specific position in the sequence of hits by Iron Byron. The distance (yards) is recorded for each of the 32 hits, and the results are shown below.

	Brand			
Club	E	F	G	H
	238.6	261.4	264.7	235.4
	241.9	261.3	262.9	239.8
Driver	236.6	254.0	253.5	236.2
	244.9	259.9	255.6	237.5
	165.2	179.2	189.0	171.4
	156.9	171.0	191.2	159.3
Five-iron	172.2	178.0	191.3	156.6
	163.2	182.7	180.5	157.4

Below is a graph of the means.



Source	df	SS	MS	F	Pr > F
Model	7	49959.3747	7137.0535	290.12	0.0001
Error	24	590.4075	24.6003		
Total	31	50549.7822			

R-Square	C.V.	Root MSE	SALES Mean
0.988320	2.3515897	4.95987	210.915625

Source	df	Anova SS	MS	F	Pr > F
CLUB	1	46443.90	46443.90	1887.94	0.0001
BRAND	3	3410.32	1136.77	46.21	0.0001
CLUBxBRAND	3	105.16	35.05	1.42	0.2600

Tukey's Studentized Range (HSD) Test for variable: DISTANCE

NOTE: This test controls the type I experimentwise error rate, but generally has a higher type II error rate than REGWQ.

Alpha = 0.1 df = 12 MSE = 24.60031

Critical Value of Studentized Range = 3.423

Minimum Significant Difference = 6.003

Means with the same letter are not significantly different.

Tukey Grouping	Mean	N	Brand
A	223.587	8	G
A	218.437	8	F
B	202.438	8	E
B	199.200	8	H

Test for a significant interact at the 10% significance level.

H_o: ______________________________

H_a: ______________________________

Pvalue: ______________________________

Conclusion: ______________________________

____

____

Test for significant main effects at the 10% significance level.

Club:

H_o: ______________________________

H_a: ______________________________

Pvalue: ______________________________

Conclusion: ______________________________

____

____

Brand:

H_o: ______________________________

H_a: ______________________________

Pvalue: ______________________________

Conclusion: ______________________________

____

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Multiple Comparison Results:

____

Conclusion: ______________________________

____

____

Complete the following ANOVA table.

Source	df	SS	MS	F	pvalue
A	4	_____	40.66	_____	_____
B	2	_____	_____	_____	_____
AxB	___	144.23	_____	_____	_____
Error	75	1206.87	_____		
Total	89	1670.28			

Test for a significant interact at the 5% significance level.

H_o: ____

H_a: ____

Pvalue: ____

Conclusion: ____

____

____

Test for significant main effects at the 5% significance level.

A:

H_o: ____

H_a: ____

Pvalue: ____

Conclusion: ____

____

____

B:

H_o: ____

H_a: ____

Pvalue: ____

Conclusion: ____

____

____

Complete the following ANOVA table.

	Source	denominato df	numerator SS	variance MS	F_{obs}	pvalue
Treatment explained	A (5)	4	162.64	40.66	2.53	0.047
	B (3)	2	156.54	78.27	4.86	0.010
	AxB	8	144.23	18.03	1.12	0.360
unexplained	Error	75	1206.87	$16.09 = s_e^2$		
	Total (90)	89	1670.28			

Test for a significant interact at the 5% significance level.

H_o: no interaction

H_a: there is a significant interaction between Factor A and B

Pvalue: 0.360

Conclusion: Fail to reject H_o, 95% sure we couldn't prove a significant interaction between Factors A and B

Test for significant main effects at the 5% significance level.

A:

H_o: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

H_a: at least 2 of 5 means differ among levels of Factor A

Pvalue: 0.047

multiple comparison test →

Conclusion: Reject H_o, 95% sure there is a difference in means between at least 2 of 5 levels of factor A

B:

H_o: $\mu_1 = \mu_2 = \mu_3$

H_a: at least 2 of 3 means differ among levels of factor B

Pvalue: 0.010

multiple comparison test →

Conclusion: Reject H_o, 95% sure there is a difference in means between at least 2 of 3 levels of factor B.